Noise Reduction in Digital IIR Filters By Finding Optimum Arrangement of Second-Order Sections

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***Abstract*-This paper proposed a new method to obtain an optimum arrangement of the second-order sections in Digital IIR Filters in order to reduce the Steady-State Output Noise Variance when the model of filter is considered as the cascade connections of second-order sections. The proposed method is based on minimizing the external normalization coefficients in each section. According to the computational complexity of obtaining the output noise variance, our method can reach to an optimal arrangement recursively, without computing the output noise variance. Experimental results show the accuracy and simplicity of our new method when it is compared with MATLAB Filter Design Toolbox (FDT).**

Keywords: *steady-state output noise variance, second-order sections, recursive method, MATLAB Filter Design Toolbox.*

I. Introduction

Input signals encountered by sonar or radar signal processing systems are continuous and must first be sampled in time and digitized in amplitude prior to processing by digital systems. Since the hardware complexity of digital signal processing systems is directly related to the digital word length, it is important to limit the number of bits in the various processing elements of the system. This requires an error analysis at each point in the system to assure that the system implementation does not degrade system performance, that is degrade dynamic range by increasing the system noise level [1].

The application of polyphase IIR structures is considered in [2] and authors show that very high performance filters can be designed by using very few coefficients. Also authors in [2] consider the different implementation structures in polyphase IIR filters and show that the effect of the quantization noise can be decreased at the time of the filter coefficient design. A new set of filter invariants called second order modes which play a definitive role in minimal noise realizations is used in [3] and authors compared its new structure with usual parallel and cascade connections of second order filters and obtained better results for narrow-band filters. In [1], authors consider the cascade second-order sections as the most common model for formation Digital IIR Filters; also they explain some of the issues related to the implementation of algorithms in special-purpose hardware. After finding zeros and poles according to characteristics of desired filter, authors in [1] describe their method for pairing (coupling) zeros and poles which is based on nearest distance between them. But in [1-6] and other previous considerations in the quantization noise issue, the effect of different arrangements in decreasing the output noise power is not focused. This paper concentrates on the arrangement of the second-order sections after coupling zeros and poles of each section according to method mentioned in [1].

The complexity of computing the output noise variance prevents examining all different arrangements. For example if an assumed filter is constructed by 5 second-order sections that the coefficients of each section are obtained by [1], 125 (5!) different arrangements can be found while the output noise variance of each arrangement is different. Therefore computing the output noise variance for all different arrangements and finding the best of them, is very time consuming and feasibly impossible.

In this paper we show that the optimum arrangement will be obtained by using a recursive algorithm which is based on minimizing the external normalization coefficients in each section without calculating the output noise variance. The optimum arrangement based on our method plays an important role in decreasing the computational complexity.

This paper is organized as follow: in section II, definitions of output noise variance, dynamic range and the external normalization coefficients are defined. In III a new viewpoint of obtaining external normalization coefficients is illustrated and the proposed algorithm is explained. Finally in section IV, the simulation results are considered to indicate the accuracy and robustness of our proposed algorithm.

II. Problem Statement

In this section first, the model of product quantization noise has been considered and then the output noise variance for cascade second-order sections will be defined according to [1].

CCECE/CCGEI May 5-7 2008 Niagara Falls. Canada
978-1-4244-1643-1/08/$25.00

As illustrated in Figure 1, if product quantization is carried out using rounding arithmetic, an error signal e(n) is added at the sum node for each multiplier. The error signal can be regarded as random process with uniform probability density function.

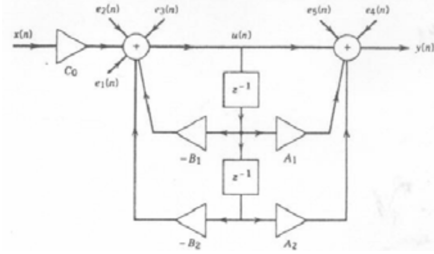

Fig1: Product Quantization Noise Model for a Section

Based on [1], for cascade sections, the noise transfer function for the noise source to the filter output is needed; that is, in general the average noise power at the k-th sections passes through all the poles and zeros of the (k+1)th and higher sections of the system. Notice that the calculation of the total computational noise generated at the output of the k-th section requires the summation of all the individual errors caused by each internal error source. The output steady-state noise variance can be formulated as:

$$\sigma_{eo}^{2} = p_n\left\{\left[r\sum_{k=1}^{M}\frac{1}{C_k^{2}}\sum_{n=0}^{N/2-1}\left|H_{k,M}(e^{j2\pi n/N})\right|^{2}\right] + \left[s\sum_{k=2}^{M}\sum_{n=0}^{N/2-1}\left|H_{k,M}(e^{j2\pi n/N})\right|^{2}\right]+s\right\} \quad (1)$$

Where

$P_n = 2^{-2b/6N}$

r=number of noise sources at the first sum node

s=number of noise sources at the second sum node

N=number of spectral points

N=frequency index

M=number of second-order sections

K=second order section index

C_k = external normalization coefficient for the k-th section

The computation of the magnitude response of the noise transfer function $H_{k,M}(e^{j2\pi\omega})$ can be obtained by using [1].

The dynamic range (DR) of a digital filter is defined as the number of available bits not corrupted by computational noise. Once the output noise variance has been determined, the DR of the filter can be expressed in decibels by

$$DR = (BIT_N - 1)20\log_{10}2 \quad (2)$$

Where BITN is the most significant bit position of the computational noise buildup given by

$$BIT_N = INTEGER\left(\frac{-\log_{10}\sigma_{eo}}{\log_{10}2}\right) \quad (3)$$

Equation (1) implies that optimal ordering of the numerator and denominator factors in the cascade structure can minimize the output roundoff noise. It can be shown that there are $M!^2$ possible ordering of second-order sections [1]. Evaluating Eq. (1) for all possible orderings can be time consuming; for example, 36 evaluations are needed for M=3, 576 evaluations for M=4. It should be noted that, however, for Butterworth and Chebyshev LP, HP and BP filters the numerator factors are respectively equal, and for these causes only M! Orderings would be necessary. It has been shown in [1] that optimal pole-zero pairing can be accomplished by pairing poles with zeros that are closest to them in z-plane. According to the above assumption, the optimum solution is obtained by first, employing the optimal pole-zero pairing, which will optimally pair the numerator and denominator second-order sections and then evaluate Eq.(1) for M! arrangements of the paired second-order sections. In this paper we propose a new method to find optimal arrangements of the paired second-order sections without using Eq(1).

III. PRPOSED METHOD

This method is based on following lemma.

Lemma: the external normalization coefficients can be obtained as follow:

$$C_i = \frac{\max(\prod_{j=1}^{i-1} H_j)}{\max(\prod_{j=1}^{i} H_j)} \quad (4)$$

$$C_0 = 1$$

Proof: consider the main formula of external normalization coefficients in [1]. According to figure 2:

$$C_i = \frac{1}{\prod_{j=0}^{i-1} C_j \max(\prod_{j=1}^{i} H_j)} \quad (5)$$

$$C_0 = 1$$

Fig2: Model of Second-Order Sections with normalization coefficients

Now obtain $C_1, C_2, \ldots$ respectively according to (5)

$$C_1 = \frac{1}{\max(H_1)} \quad (6)$$

$$C_2 = \frac{1}{C_1\max(H_1H_2)} \quad (7)$$

By replacing (6) in (7)

$$C_2 = \frac{1}{C_1\max(H_1H_2)} = \frac{1}{\frac{1}{\max(H_1)}\max(H_1H_2)} = \frac{\max(H_1)}{\max(H_1H_2)}$$

$$C_3 = \frac{1}{C_1C_2\max(H_1H_2H_3)} \quad (8)$$

And by replacing (6),(7) in (8) we will have

$$C_3 = \frac{1}{C_1C_2\max(H_1H_2H_3)}$$

$$=\frac{1}{\frac{1}{\max(H_1)}\frac{\max(H_1)}{\max(H_1H_2)}\max(H_1H_2H_3)}=\frac{\max(H_1H_2)}{\max(H_1H_2H_3)}$$

And it can be shown that for C_4:

$$C_4=\frac{1}{C_1C_2C_3\max(H_1H_2H_3H_4)}=\frac{\max(H_1H_2H_3)}{\max(H_1H_2H_3H_4)}$$

$\vdots$

$\vdots$

$$C_i=\frac{1}{\max(\prod_{j=0}^{i-1}C_j)\max(\prod_{j=1}^{i}H_j)}=\frac{\max(\prod_{j=1}^{i-1}H_j)}{\max(\prod_{j=1}^{i}H_j)}$$

The stages of our proposed method which are based on minimizing C_i in each section can be summarized as follows:

A. Assume M second-order sections for a specific filter are needed which numerators and denominators of each section are paired according to previous assumptions. Now consider the last section and last coefficient. If H_i is the last section, then

$$C_{M=}\frac{\max(\prod_{j=1}^{j=M-\{i\}}H_j)}{\max(\prod_{j=1}^{j=M}H_j)}$$

As it can be seen, the denominator of C_M is constant and independent of all different arrangements. So in order to minimize the C_M, it is enough to minimize the numerator of C_M , therefore **M different states** should be examined to find the minimum of the numerator. As an example assume a filter contains **3 second order sections** namely H_1 H_2 H_3 , so in this step the last section or **3th** section will be defined as follow:

1-if the H_1 is the last section then

$$C_{3=}\frac{\max(H_2H_3)}{\max(H_1H_2H_3)}$$

2- If the H_2 is the last section then

$$C_{3=}\frac{\max(H_1H_3)}{\max(H_1H_2H_3)}$$

3- If the H_3 is the last section then

$$C_{3=}\frac{\max(H_1H_2)}{\max(H_1H_2H_3)}$$

After comparing these 3-states and finding the **minimum state**, then the last section will be chosen.

B. after obtaining the last section (assume H_i), the algorithm is going to obtain the **(M-1)th** section.

$$C_{M-1=}\frac{\max(\prod_{j=1}^{j=M-\{i,k\}}H_j)}{\max(\prod_{j=1}^{j=M-\{i\}}H_j)}$$

Assume H_k is pointed to **(M-1)th** section then According to previous step (finding the last section) the denominator of C_{M-1} will be constant and **(M-1) different states** should be examined for finding the minimum of numerator. Assume H_k is the **(M-1)th** section and $H_w\neq\{H_i,H_k\}$. Following **inequality** should be considered in order to find (M-1)th section according to previous assumption which was based on minimizing the numerator in each section:

$$\max(\prod_{j=1}^{M-\{i,k\}}H_j)<\max(\prod_{j=1}^{M-\{i,w\}}H_j)\ \text{ for all }\ H_w\neq\{H_i,H_k\}$$

C. Continue this method to obtain H_1 as the first section.

Figure (3) shows the schematic of proposed algorithm.

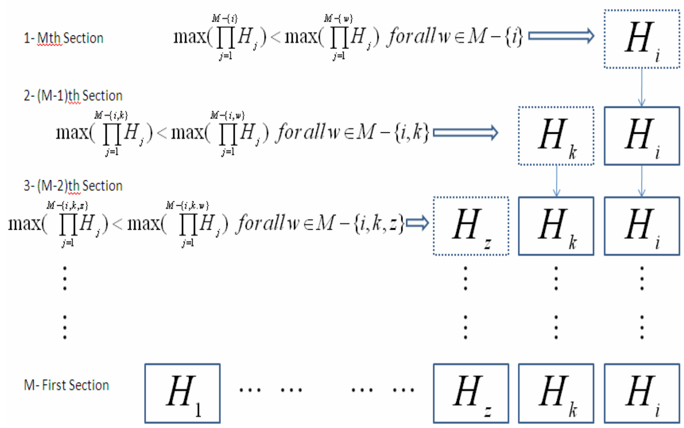

Fig3: Proposed Algorithm

IV. Simulation Results

In this part, 2 different types of filters are considered. In all of them, the numerators and denominators are obtained and paired by using MATLAB Filter Design Toolbox (number of bits=16).

Experiment 1: A Band Stop Chebychiev(type 1) IIR filter with fpass1=60Hz, fstop1=120Hz, fstop2=180Hz, fpass2=240Hz, $Apass_{1,2}$=0.5dB, Astop=35dB, sampling frequency =4000.

Table (1) shows the coefficients of the denominator and numerator of section one, H1, to section 5, H5, by using MATLAB filter design toolbox. The effect of the different arrangements of the second order sections in the output noise variance and output dynamic range is shown in table (2). According to that, the output steady state noise variance which is obtained by our proposed algorithm is **2.7826e-009** while it is 1.6499e-008 when FDT is considered. As can be seen, the output noise variance which is obtained by our algorithm is approximately 6 times lower than that when FDT algorithm is used.

Table 1: Designing filter (experiment 1) by using MATLAB Filter Design Toolbox

Section i	Numerator	Denominator
H1	N1= [1.0000 -1.9643 1.0000]	D1 = [1.0000 -1.8195 0.9533]
H2	N2= [1.0000 -1.9643 1.0000]	D2 = [1.0000 -1.5624 0.7481]
H3	N3= [1.0000 -1.9643 1.0000]	D3 = [1.0000 -1.9785 0.9875]
H4	N4= [1.0000 -1.9643 1.0000]	D4 = [1.0000 -1.9468 0.9524]
H5	N5= [1.0000 -1.9643 1.0000]	D5= [1.0000 -1.4103 0.4360]

Table 2: The effect of Different Arrangements in output noise variance and the dynamic range

Algorithms	Arrangement	Output Steady-State noise Variance	Dynamic Range (DR)
MATLAB FDT	{H1 H2 H3 H4 H5}	1.6499e-008	66.2266 dB
Random Arrangement	{H2 H1 H4 H5 H3}	9.0716e-008	60.2060 dB
Proposed Method	**{H2 H4 H5 H1 H3}**	**2.7826e-009**	**78.2678 dB**

Experiment 2: A Band Stop Eliptic IIR filter with fpass1=72Hz, fstop1=96Hz, fstop2=1200Hz, fpass2=1440Hz, $Apass_1$=0.5dB, Astop=60dB, $Apass_2$=1dB, sampling frequency =4000.

Table (3) shows the coefficients of the denominator and numerator of section one, H1 to section 7, H7 by using MATLAB filter design toolbox (FDT). The effect of different arrangements of the second order sections in the output noise variance is shown in table (4). According to that, the output steady state noise variance which is obtained by our proposed algorithm is **5.2781e-006** while it is 4.0966e-005 when FDT is considered. The output noise variance which is obtained by our algorithm is approximately 8 times lower than that is obtained by using FDT algorithm.

Table3: Designing filter (experiment 2) by using MATLAB Filter Design Toolbox

Section i	Numerator	Denominator
H1	N1=[1 0.0405 1]	D1=[1 -1.9393 0.9443]
H2	N2=[1 -1.9451 1]	D2=[1 1.3711 0.6414]
H3	N3=[1 0.8049 1]	D3=[1 1.2744 0.8343]
H4	N4=[1 -1.9754 1]	D4=[1 -1.9644 0.9747]
H5	N5=[1 1.0076 1]	D5=[1 1.2405 0.9557]
H6	N6=[1 -1.9809 1]	D6=[1 -1.9805 0.9933]
H7	N7=[1 -1.5704 1]	D7=[1 -0.2434 - 0.6899]

Table 4: The effect of Different Arrangements in output noise variance and the dynamic range

Algorithms	Arrangement	Output Steady-State noise Variance	(DR)
MATLAB FDT	{H1 H2 H3 H4 H5 H6 H7}	4.0966e-005	36.1236 dB
Random Arrangement	{H5 H7 H4 H1 H3 H6 H2}	2.8488e-004	24.0824 dB
Proposed Method	**{H2 H1 H5 H6 H3 H4 H7}**	**5.2781e-006**	**42.1442 dB**

Also the results in both experiments show that the dynamic ranges obtained based on our proposed algorithm are higher than they when MATLAB (FDT) and random arrangement are considered.

V. **Conclusion**

A novel method for noise reduction in Digital IIR Filters by finding optimal arrangement of second-order sections has been proposed in this paper. The experimental results show the robustness and precision of proposed method in comparison with MATLAB Filter Design Toolbox. It is shown that our method not only is easy to implement but also has lower complexity. Moreover the number of states which is needed for finding the optimum arrangement is much lower than previous works. According to the encouraging results, this method can be used in DSP processors and FPGA applications.

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