Deconvolution of non-Minimum Phase FIR Systems Using Recursive Genetic Algorithm

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***Abstract*-the problem of estimating the input sequence of a known, non-minimum phase system is considered in this paper. The proposed method is based on minimizing the sum of squared differences between the original and the estimated output. The estimated output is obtained by exciting the system with un-known input signal which begins with initial values and is updated step by step in order to minimize the mentioned error. New viewpoint of the convolution equation allows to: 1) identify the un-known parameters of the input sequence recursively and 2) apply any optimization algorithm in the deconvolution problem. Genetic Algorithm optimization is considered in this paper because of its power in searching the entire solution space with more probability of finding the global optimum. This approach covers the deconvolution of the both FIR and IIR non-minimum phase filters. Also simulation results show the accuracy and simplicity of our proposed algorithm in the deconvolution of the non-minimum phase, high order FIR filters common in seismic and speech signal processing.**

***Keywords**: Recursive Genetic Algorithm, Non-Minimum Phase Systems, Deconvolution.*

I. Introduction

According to [1], the deconvolution problem is intricate for at least two reasons: the measurements are usually noise corrupted, and the system is frequently non-minimum phase. These problems restrict the use of a simple deconvolution filter, namely, the inverse system. These restrictions placed on the inverse system filter design depend on the application. The main problem derives from inversing non-minimum phase systems which makes systems to be unstable (some poles are located outside the unit circle in the z-plane). There is a wide range of applications, including seismology, equalization, numerical differentiation, and speech synthesis. See, for example, [1], [2], [9],[4], [7], [5], [10], and [11]. The list can be much longer for what is the common interest-estimating the input to a LTI system.

In [1], authors approached the deconvolution problem from a shift operator point of view, seeing it as a linear quadratic optimization problem.

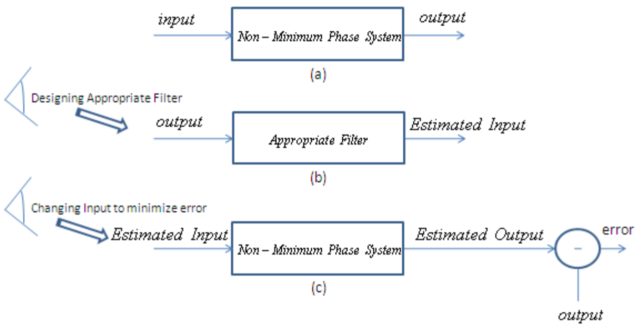

Fig1: a) Non-minimum phase system. b) Viewpoint of previous works in deconvolution problem that causes to design appropriate filter reach to input signal. c) Viewpoint of proposed method which.

This approach covers input prediction, filtering and smoothing problems, and the use of pre-filters in the quadratic criterion. Such problems can be approached with different methods such as Kalman filtering [3], Wiener filtering [8], or Wiener optimization of filters with predetermined structure, such as FIR filters [6].

In this paper we will approach the deconvolution problem from a new viewpoint of the convolution equation in noise free, FIR systems. Also, the proposed algorithm will be expanded to cover IIR (ARMA model) filters. Our perspective of the convolution equation causes to find a recursive optimization algorithm and identify the un-known parameters of the input signal step by step. Any optimization method can be used in our algorithm, but GA (Genetic Algorithm) is more important in comparison with conventional optimization methods because of its power in searching the entire solution space with more probability of finding the global optimum. We do, however, believe that the solution presented here provides important insights, not only, in the estimating the input signal of a non-minimum phase system, but also, in using recursive optimization methods without consideration of the length of the un-known parameters which is the serious problem of all optimization algorithms.

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Figure (1.a) shows the general illustration of non-minimum phase, LTI systems. Figure (1.b) shows the view point of previous works in the deconvolution problem and figure (1.c) shows a new point of view which is the basis of this paper. The organization of this paper is as follows: in section II, definitions of deconvolution problem are defined and a new viewpoint of obtaining the parameters of the input sequence is shown. In section III the proposed algorithm is explained and finally in section IV, the simulation results are considered to indicate the accuracy and robustness of our proposed algorithm.

II. Problem Statement

Consider a linear time invariant system which is assumed to be non-minimum phase FIR. This system can be described by convolution formula.

$$y(n) = b(n) * h(n) = \sum_{i=0}^{q} b(n-k)h(k) \quad (1)$$

$$w(n) = y(n) + v(n) \quad (2)$$

Where $b(n)$ is the unknown input sequence (in the deconvolution problem), $h(n)$ is the finite impulse response shown as $\{h(n)\}_{n=0}^{q}$ that is non-minimum phase, $\{y(n)\}_{n=0}^{L}$ is the noise free output, $v(n)$ is a additive, zero mean, Gaussian noise and $w(n)$ is the noisy output. Figure (2) describes the model.

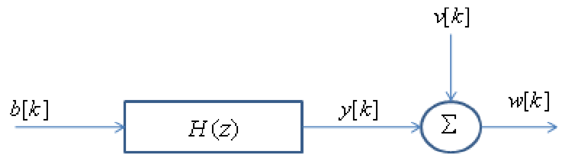

Fig2: Model of a LTI system

Assume that $h(n)$ and $y(n)$ are available. In this part, in order to simplify the description of our model, we focus on noise free system. By expanding equation (1), the following relations are obtained.

$$\begin{aligned} &y(0) = b(0)h(0) \\ &y(1) = b(0)h(1) + b(1)h(0) \\ &y(2) = b(2)h(0) + b(1)h(1) + b(0)h(2) \\ &\vdots \\ &y(n) = \sum_{k=0}^{q} b(n-k)h(k) \end{aligned} \quad (3)$$

The relations in equation (3) show that unknown input sequence $b(n)$ can be obtained recursively which is the basis of our proposed method. Consider figure (3) which illustrates our perspective in solving unstable deconvolution problems.

In this model, sequence $\{b(n)\}_{n=0}^{L}$ is the unknown input signal that must be found by minimizing the sum of differences between the output, $y(n)$, and its estimation $\hat{y}(n)$. The result of exciting the non-minimum phase system with the unknown sequence $\{b(n)\}_{n=0}^{L}$ is $\hat{y}(n)$. This new model allows us to consider the non-minimum phase system as any other system without inversing it which causes instability. The fitness function is as follow:

$$J = \sum error^2(n) = \sum_{n=0}^{L} [y(n) - \hat{y}(n)]^2 = \sum_{n=0}^{L} [y(n) - \sum_{k=0}^{q} b(n-k)h(k)]^2 \quad (4)$$

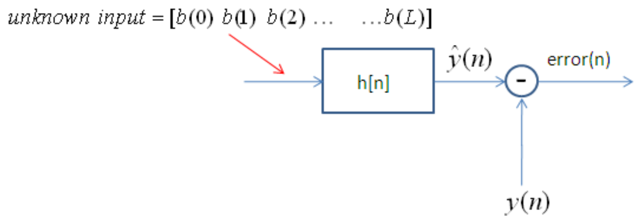

Fig3: Model of the deconvolution problem

The main issue in finding the unknown sequence $\{b(n)\}_{n=0}^{L}$ is its length L that can make any optimization method go wrong. The disability of optimization methods increases rapidly with the length of signal. We solved the above problem by using recursive optimization method that permits to obtain parameters step by step. The results are independent of the length of input or output signal. Consider figure (4) that allows us obtain the parameters of the input signal recursively. Note that analytical solution for equation (3) exists, but the input sequence which is obtained by analytical solution, diverges to infinity because of the error build-up.

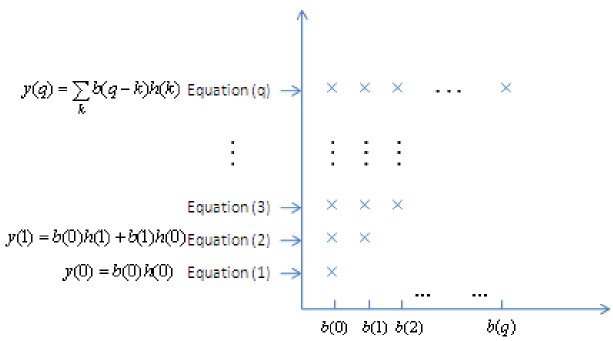

Fig4: Relations between input parameters and partial convolution equations

Figure (4) illustrates how each input parameter appears in different convolution value as described in (3). It shows all q equations (as assumed) related to $b(0)$, also (q-1) equations related to $b(1)$ and so on. Therefore, on the basis of their representations in different equations, it seems logical to start estimating $b(0)$ first, followed by $b(1)$ and so on in a recursive manner. As we use an optimization procedure that takes into

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account as many as possible convolution values for the estimation of each input parameter, the error is not built up as in the iterative analytical solution.

III. PRPOSED ALGORITHM

This algorithm is based on the recursive relation of parameters shown in equation (3). The model of our algorithm is demonstrated in figure (2). Now consider the following algorithm.

Algorithm:

1. Initial value

Set b =zeros(1,L), where L is the length of output signal (equals to the length of the input signal).

2. Finding first 5 parameters of sequence b

Find x(0), x(1), x(2), x(3), x(4) by minimizing equation (4), using genetic algorithm optimization. Consider figure (5) which shows the model of finding first 5 parameters of the input sequence b .

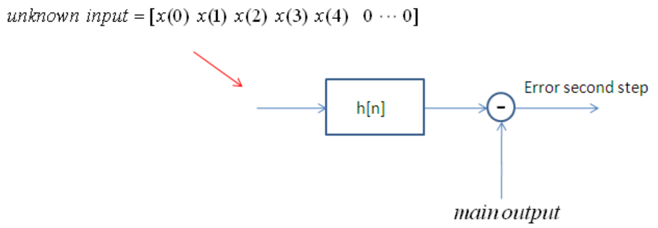

Fig5: Model of finding first 5 parameters of sequence b

After this step and determination the unknown parameters using genetic algorithm optimization, b(0)=x(1), b(1)=x(2), b(2)=x(3), b(3)=x(4), b(4)=x(5). The sequence of input signal b will be $b = [b(0)\, b(1)\, b(2)\, b(3)\, b(4)\ 0 \ldots 0]$.

3. Finding second 5 parameters of sequence b

Find x(0), x(1), x(2), x(3), x(4) similar to step 2 as the second set of 5 unknown parameters. Figure (6) illustrates the model of finding second 5 parameters of sequence b . Consider that parameters b(0:4) are determined in the second step and x(0:4) are unknown parameters that should be estimated in this step.

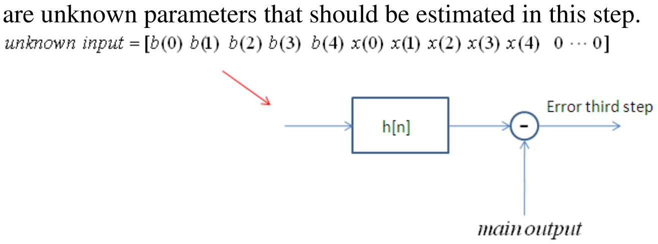

Fig6: Model of finding second 5 parameters of sequence b

After this step and determination of unknown parameters using genetic algorithm optimization, b(5)=x(0), b(6)=x(1), b(7)=x(2), b(8)=x(3), b(9)=x(4). The sequence of input signal b will be:
$b = [b(0)\, b(1)\, b(2)\, b(3)\, b(4)\, b(5)\, b(6)\, b(7)\, b(8)\, b(9)\ 0 \ldots 0]$.

4. Continue to find all parameters

Similar to steps 2 and3, in each step estimate b(k), b(k+1), b(k+2), b(k+3), b(k+4).

Note1: the convergence of the proposed algorithm is confirmed in practice and is intuitively explained on the basis of equation (3) and figure (3) considered in the previous section.

Note2: this algorithm can be repeated after finding all parameters of sequence b then used as initial values.

GA Parmeters:

The genetic algorithm is a stochastic optimization algorithm that was originally motivated by the mechanisms of natural selection and evolution of genetics. In the following, a parameter estimation algorithm is developed based on GA to estimate the unknown parameters, by carrying out minimization of the sum squared errors in equation (4).

All GAs are effective when used with its best operations and values of parameters [13]. The following parameters are modified due to experimental results.

1) The fitness function is considered in (4) that must be minimized. For standard optimization algorithm this is known as the objective function.

2) The population size determines the size of the population at each generation. Choosing the population size as 20 will be satisfied the results. This optimization includes five variables so the population can be represented with a 20 by 5 matrix. At each iteration, the genetic algorithm performs a series of computation on the current population to produce a new population. The algorithm begins by creating a random initial population in the interval of [-1, 1].

3) At each step, the genetic algorithm uses the current population to create the children that make up the next generation. Algorithm usually selects individuals that have better fitness value.

“Stochastic uniform” used as selection mechanism; it is robust and simple.

4) “Elite” children are the number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. Elite count considered in this paper is 5 or 25% of population size.

The “termination criterion” is reaching at 100th generation that means algorithm is repeated until the number of generations equal to 100.

5) “Gaussian mutation” and “heuristic crossover” are used to produce offspring for the next generation. As known a Gaussian mutation operator requires two parameters: the mean, which is often set to zero, and the standard deviation δ . δ =5.5 is satisfactory in this optimization. Suppose a child is

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considered as a value will be produced in the next generation and parents are the values which obtained in previous generation, "heuristic crossover" returns a child that lies on the line containing the two parents, a small distance away from the parent with the better fitness value in the direction away from the parent with the worse fitness value. The parameter Ratio can specify how far the child is from the better parent. The following equation illustrates the relation between parameter Ratio and child (as next generation).

Child= parent 2 + R (parent 1- parent 2)

Where parent 1 & parent 2 are the current generating parents, and of course parent 1 has the better value, and R is the parameter Ratio. R=1.2 is employed.

6) Using "Hybrid function" increases the robustness of genetic algorithm. A hybrid function is an optimization function that runs after the genetic algorithm terminates in order to improve the value of fitness function. The Hybrid function uses the final point from the genetic algorithm as its initial point.
We use the function "fminsearch" an un constrained minimization function in the MATLAB optimization toolbox. "fminsearch" uses the simplex search method of [12]. This is a direct search method that does not use numerical or analytic gradients.

IV. SIMULATION RESULTS

In the following examples, we use our recursive algorithm to identify the input sequence from output data and the assumed non-minimum phase (NMP) system. At the end of this part, by comparing original and the estimated input, the accuracy of our proposed algorithm will be shown.

Example 1: (NMP-MA case) the original input signal that should be estimated is shown in figure (8, left). The relation of the output and input is as follow:

$$\begin{aligned}y(i) &= 10x(i)+20x(i-1)+31x(i-2)-40x(i-3)\\&\quad -22x(i-4)+31x(i-5)+10x(i-6)-42x(i-7)\\&\quad +31x(i-8)+12x(i-9)-30x(i-10)+14x(i-11)\\&\quad -23x(i-12)+56x(i-13)+9x(i-14)-2x(i-15)\end{aligned}$$

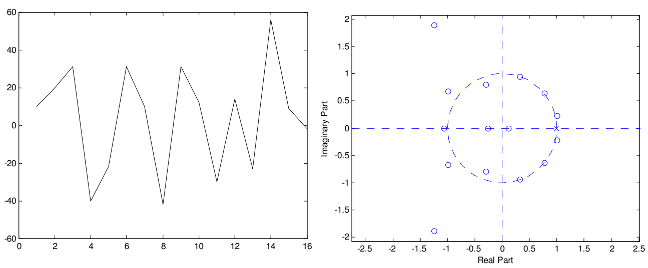

Fig7: Left: the impulse response of system, Right: pole-zero plot

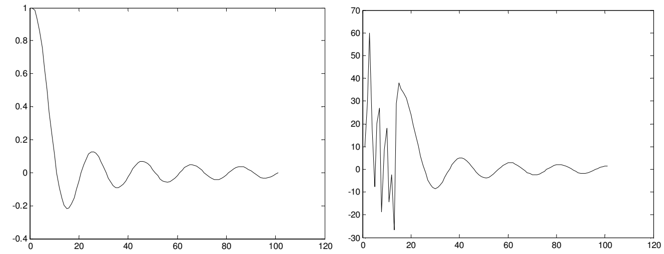

Fig8: Left: the original input, right: The output of system

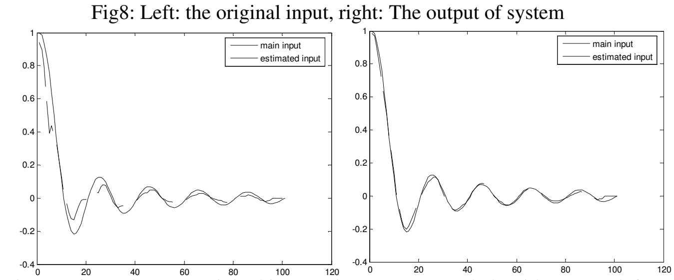

Fig9: Estimated versus original input using our proposed algorithm once (left) and for three times (right).

Figure7 shows the impulse response of assumed system and the zeros location of its transfer function in z-domain respectively. Also figure9 shows the estimated sequence of input signal using our proposed algorithm respectively once and three times which shows the accuracy of our proposed algorithm in the deconvolution of systems that are non-minimum phase (un-stable due to inverse filtering).

Deconvolution of NMP-ARMA Model System

It is obvious that in this case, the relation of parameters in the convolution equation will be changed because of the AR part. One way is separating the MA and AR parts and using the previous algorithm that is considered in figure (4). Consider the ARMA model (figure (10)).

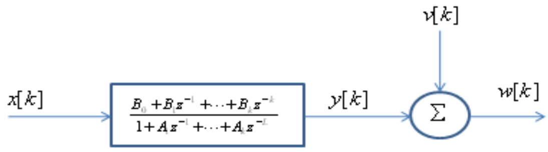

Fig10: the ARMA Model

$$\begin{aligned}y(n) &= -A_1 y(n-1)-\cdots - A_L y(n-L)+B_0 x(n)+B_1 x(n-1)\\&\quad +\cdots + B_k x(n-k)\end{aligned} \tag{5}$$

Where the system ($\dfrac{B(z)}{A(z)}$) and the output ($y(n)$) are available and it is assumed that the system is noise free. In order to use our proposed algorithm, equation (5) should be converted to an MA model. Therefore we use the following model.

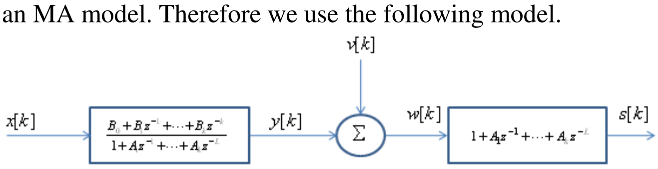

Fig11: Convert ARMA model to MA model

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$$s(n) = x(n) * B(n) = \sum_{i=0}^{k} x(n-i)B(i) \qquad (6)$$

$$s(n) = y(n) + A_1 y(n-1) + \cdots + A_L y(n-L) = B_0 x(n) + B_1 x(n-1) + \cdots + B_k x(n-k)$$

Example 2: (NMP-ARMA case) the true input signal that should be estimated is shown in figure (13, left), and the relation between the output and input is as follow:

$$\begin{aligned} y(i) &= 1.562y(i-1) - 0.496y(i-2) + 0.2601y(i-3) \\ &- 0.0975y(i-4) - 0.2310y(i-5) + 0.0001y(i-6) \\ &- 0.0521y(i-7) - 14.56y(i-8) + 11.06y(i-9) \\ &+ 0.0870y(i-10) + x(i) + 2x(i-1) + 3x(i-3) \\ &- 4x(i-4) + 5x(i-5) - 2.1x(i-6) + 3x(i-7) \end{aligned}$$

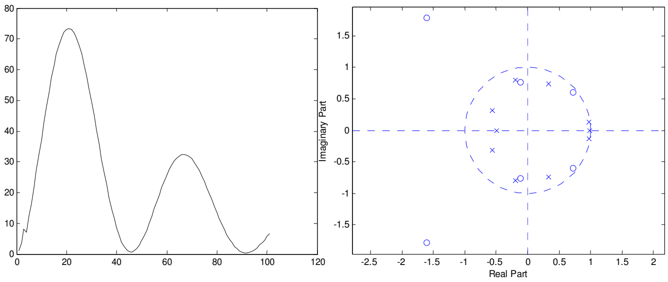

Fig12**:** Left: the impulse response of system, Right: pole-zero plot

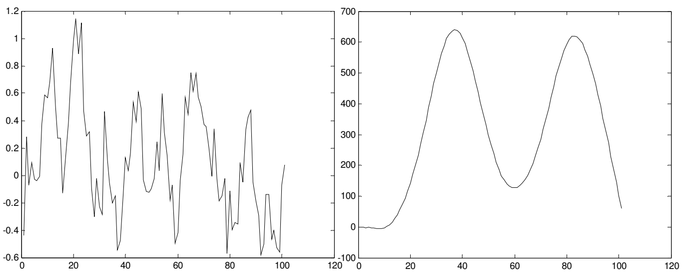

Fig13: Left: the original input, Right: the output of system

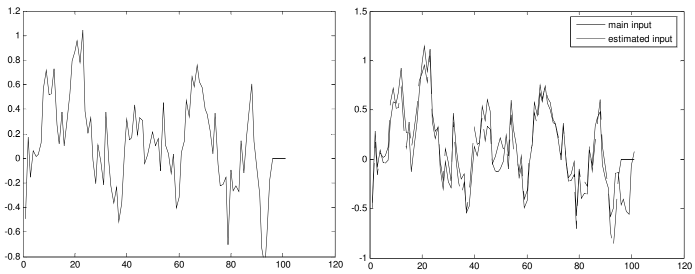

Fig14: Left: the estimated input (using algorithm for three times), Right: the original versus estimated input

Figure12 shows the impulse response of the system and the pole-zero locations of its transfer function respectively. Figure14 (left) shows the estimated sequence of the input signal using our proposed algorithm for three times, also figure14 (right) indicate the estimated versus original input.

V. Conclusion

We described a novel method for deconvolution of non-minimum phase systems. We proposed a new perspective on convolution formula which leads to a recursive algorithm for obtaining the parameters of the input sequence. Unlike previous studies our approach does not cater for designing a deconvolution filter in order to estimate the input. Therefore, the proposed algorithm can be used in non-minimum phase systems as well as minimum phase systems. Also our approach covers both MA and ARMA models. Moreover the results show the accuracy and robustness of our proposed method in identifying the parameters of the input sequence where the assumed system is non-minimum phase.

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