HOS-Based Non-Minimum Phase MA Parameter Estimation using Genetic Algorithm

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***Abstract*-In this paper we present a novel method for identification of linear time invariant, non-minimum phase (NMP), FIR systems, when only output data are available and the order of filter is higher than four. We generally model a non-minimum phase system as an MA model of known order. To estimate the parameters of our model, we exploit 1-D diagonal slice of third order cumulant of output, which may be contaminated by additive, zero mean, Gaussian white noise of unknown variance. This method is based on a new point of view on the third order cumulant equation and using recursive optimization method in order to identify un-known parameters by minimizing the sum of squared differences between the observed cumulant (diagonal slice) and the cumulant of proposed model. We propose both analytical and optimization-based solutions for identifying the filter coefficients and using analytical-based solution as a view point of applying our recursive optimization algorithm which causes to obtain parameters recursively. Also Genetic Algorithm optimization is considered as optimization method in this paper. Moreover Experimental results indicate the robustness and accuracy of proposed algorithm for high order systems.**

***Keywords**: Higher Order Statistics (HOS), MA Parameter Estimation, Recursive Algorithm, Genetic Algorithm Optimization.*

I. INTRODUCTION

During recent years higher order statistics (spectra) have begun to find wide applicability in many diverse fields; e.g., sonar, radar, plasma physics, biomedicine, seismic data processing, image reconstruction, harmonic retrieval, time delay estimation, adaptive filtering, array processing, blind equalization and blind system identification[1]. These statistics, known as cumulants, and their associated Fourier transforms, known as poly spectra, not only reveal amplitude information about a process, but also reveal phase information. This is important, because, as is well known, second order statistics are phase blind.

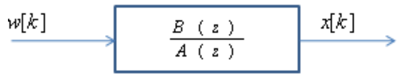

Fig1: Model Transfer Function

Various recursive and least squares methods for the identification of MA systems have been proposed using a variety of second, third and fourth order statistics and different 1-D cumulant slices [3], [5], [9], [10-11]. Mendel in [1] categorized the methods for blind MA system identification in three groups. 1) Closed-form solutions. 2) Linear algebra solutions. 3) Optimization solution.
[4] developed a new method for MA parameter estimation which exploited all samples of the second and third order cumulants to reconstruct the unknown system impulse response. [6] and [8] proposed methods that depend on third order cumulants alone. Method [7] is remarkably different as it uses a combination of second, third and fourth order statistics. This aim in using this combination of cumulant orders is to support intrinsically and automatically all non-Gaussian input distributions. Hence method [7] can be used more widely and with more confidence regardless of prior knowledge of the system statistics. For an MA process the system output is related to the input by the convolution sum of the input with the rational system transfer function, $B(z)$ figure (1). From other prospective, two techniques are employed in the estimation of the filter coefficients: nonlinear methods and linear simplifications. Nonlinear solutions can be computationally expensive and may converge to local minimum [4]. However, when these nonlinear methods are properly initialized, the estimates obtained are generally better than the estimates obtained using linear methods. MN96 is a nonlinear method which currently uses the $c(q,k)$ formula [9] to initialize the algorithm. These mentioned methods assume prior knowledge of MA model order, q. In fact, estimation of q from the time series is a substantial part of the system identification problem. Hence, model order selection has become an area of research in its own right.

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Up to now, researches in ([12], [13], [14], [15], [16]) have concentrated mostly on identifying MA filter coefficients with the order lower than five. The main problem of HOS-based MA parameter estimation for high order filters is due to propagation errors in estimating the parameters which is derived from the high nonlinearity of cumulant-based equations due to parameters. This problem is solved by proposed method. The power of proposed method in estimating the parameters of systems with the model order higher than 4, derives from the recursive characteristic of our method.

Organization of this paper is as follows: in section II, definitions of MA filters and third order cumulant equation are defined, in section III, the problem of MA parameter estimation using third order cumulants is stated and in section IV, a new viewpoint of reaching to MA filter coefficients is shown and the proposed algorithm is explained in section V, and finally in section VI, the simulation results are considered to indicate the accuracy and robustness of proposed algorithm.

II. MA FILTERS AND HIGHER ORDER STATISTICS

Following [3], consider the system depicted schematically in figure (2). The noise-free signal, $x[k]$, is related to the driving noise, $w[k]$, by parameters $b[i]$ where i takes the values 0,1, $\cdots$, q and q is the model order. The observed output, $y[k]$, is corrupted by additive Gaussian noise, $v[k]$. Figure (3) shows a more generic representation of the MA(q) process where the filtering operation is depicted by the function B(z). For an MA process B(z) is rational polynomial with order q equal to the number of previous inputs that affect the current output which gives the order of the process. The noise-free output, $x[k]$, is related to the input, $w[k]$, by constant weights, $\{b(i)\}_{i=0}^{q}$, given in equation (1). The corruption of this output by noise results in equation (2) where the observed noise-corrupted signal, $y[k]$, is summation of the noise-free signal and the additive noise $v[k]$.

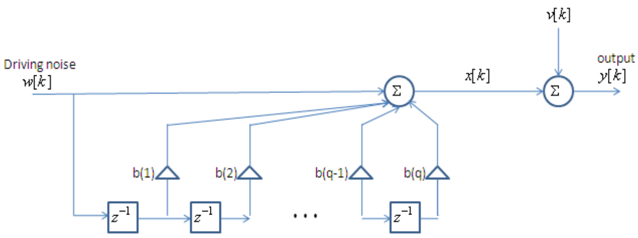

Fig2: MA Modeling

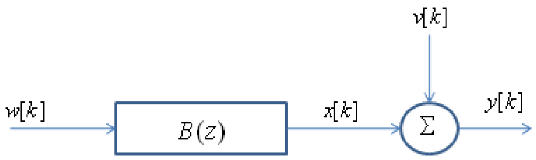

Fig3: Notation for Cumulant Identification Methods

$$x[k] = \sum_{i=0}^{q} b(i) w[k-i] \tag{1}$$

$$y[k] = x[k] + v[k] \tag{2}$$

If higher order statistics are to be used to formulate a general relationship for the identification of the model parameters from the output of the system only, then the following conditions 1 and 2 must hold:

1. The driving noise, $w[k]$, is zero-mean, independent and identically distributed (i.i.d) and non-Gaussian with $E\{w^2[k]\} = \gamma_{2w}^2$, $E\{w^3[k]\} = \gamma_{3w} \neq 0$, and $E\{w^4[k]\} - 3\gamma_{2w} = \gamma_{4w} \neq 0$.
2. The measurement noise, $v[k]$, is assumed to be zero-mean, i.i.d., and independent of $w[k]$. In addition it is assumed to be Gaussian in distribution with $E\{v^2[k]\} = \sigma_v^2$, $E\{v^3[k]\} = \gamma_{3v} = 0$, $E\{v^4[k]\} - 3\sigma_v^2 = \gamma_{4v} = 0$.

Let $c_{3x}(m,n)$ represent the third order cumulant of the observed noise-free signal at lags m and n. equation (3) relates the third-order cumulant at the specified combination of lags to the MA parameters $\{b(i)\}_{i=0}^{q}$ and the skewness of the input to the system, γ_{3w}.

$$c_{3x}(m,n) = \gamma_{3w} \sum_{k=0}^{q} b(k) b(k+m) b(k+n) \tag{3}$$

This relation is the basis for all methods of blind system identification which make use of third-order statistics.

III. PROBLEM STATEMENT

Equation (3) can be modified to yield an equation using the diagonal slice of the third order cumulants. This method is obtained by setting m=n in (3) generating a relation between the diagonal cumulant slice $c_{3x}(k,k)$ and the MA parameters to be estimated [3].

$$c_{3y}(m,m) = \gamma_{3w} \sum_{k=0}^{q} b(k) b(k+m)^2 \qquad m = -q, \ldots, 0, \ldots, q \tag{4}$$

The fitness function which should be minimized according to equation (4) is as follow:

$$J = \sum_{m=-q}^{q} [c_{3y}(m,m) - \gamma_{3w} \sum_{k=0}^{q} b(k) b(k+m)^2]^2 \tag{5}$$

[17] suggested two methods for the estimation of the MA coefficients based on the $c_{3x}(k,k)$ formula: a nonlinear least-square approach, and a linear programming approach. The computational overhead in [17] is very high and rapidly increase with model order, and similar to other previous works, is only appropriate for low order filters ($q \leq 4$).

We present a novel point of view in equation (3) that allows us obtain unknown parameters recursively. We will show next

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the analytical basis of our optimization method to estimate the parameters of a non-minimum phase MA filter.

IV. ANALYTICAL VIEWPOINT

According to equation (3), following equations will be obtained:

$$\begin{cases} c_{3y}(q,q) = \gamma_{3w} b(0) b(q)^2 \\ c_{3y}(-q,-q) = \gamma_{3w} b(0)^2 b(q) \end{cases} \tag{6}$$

$$\begin{cases} c_{3y}(q-1,q-1) = \gamma_{3w}[b(0)b(q-1)^2 + b(1)b(q)^2] \\ c_{3y}(-(q-1),-(q-1)) = \gamma_{3w}[b(0)^2 b(q-1) + b(1)^2 b(q)] \end{cases} \tag{7}$$

$$\begin{cases} c_{3y}(q-2,q-2) = \gamma_{3w}[b(0)b(q-2)^2 + b(1)b(q-1)^2 \\ \qquad + b(2)b(q)^2] \\ c_{3y}(-(q-2),-(q-2)) = \gamma_{3w}[b(0)^2 b(q-2) + b(1)^2 b(q-1) \\ \qquad + b(2)^2 b(q)] \end{cases} \tag{8}$$

$$\vdots$$

$$\vdots$$

$$c_{3y}(0,0) = \gamma_{3w}(b(0)^3 + b(1)^3 + \cdots\cdots + b(q)^3) \tag{9}$$

It can be seen from the system of equations (6, 7, 8) that: $b(0)$, $b(q)$ can be obtained by solving Eq(6). $b(1)$, $b(q-1)$ Will be obtained next by using $b(0)$, $b(q)$ and solving Eq(7). $b(2)$, $b(q-2)$ will be estimated using $b(0)$, $b(q)$ and $b(1)$, $b(q-1)$ and solving (8) by the same token. Other parameters are defined in the same way recursively.

V. OPTIMIZATION-BASED SOLUTION

In this part, we describe our algorithm which is based on recursive estimation of unknown parameters. Consider the following model.

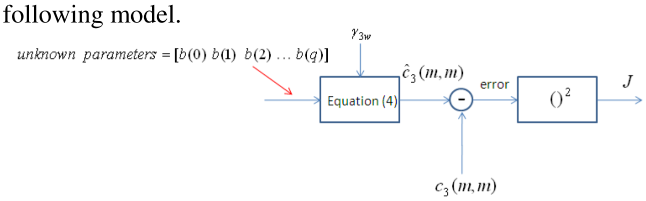

Fig3: Model of proposed algorithm

Where γ_{3w} is the skewness of input signal which can be calculated using the method in [2], $c(m,m)$ is the 1-D diagonal slice of third order cumulant of output, and, $\hat{c}(m,m)$ is available using equation (4) and the sequence of b are unknown parameters which should be determined by minimizing equation (5). We propose following algorithm which is based on recursive dependence of parameters to each other considering equations (6-9), in order to find the sequence of b (MA filter coefficients).

Proposed Algorithm

Step1.initial value

Set b =zeros(1,q), where q is the order of filter (in this paper, it is assumed that the order of filter is definite).

Step2.finding b(0) and b(q)

Insert x(1)=b(0) and x(2)=b(q) and find x(1) and x(2) by minimizing equation (5) using genetic algorithm optimization. Consider figure (4) which shows the model of finding b(0) and b(q).

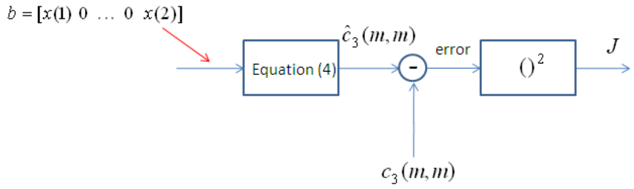

Fig4: Model of second step of proposed algorithm

After this step b(0)=x(1), b(q)=x(2). The sequence of MA filter coefficients (b) will be $b = [b(0)\ \ 0 \ldots 0\ b(q)]$.

Step3.finding b(1) and b(q-1)

Using b(0) and b(q) from step 2 and insert x(1)=b(1) and x(2)=b(q-1), then find x(1) and x(2) by minimizing equation (5), using genetic algorithm optimization. Consider figure (5) which shows the model of finding b(1) and b(q-1).

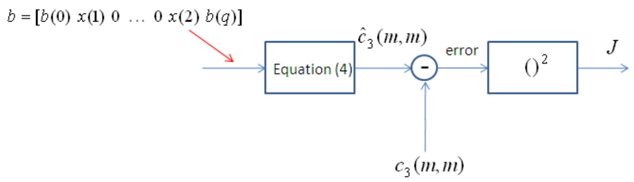

Fig5: Model of third step of proposed algorithm

After this step b(1)=x(1), and b(q-1)=x(2). The sequence of MA filter coefficients (b) will be as follow: $b = [b(0)\ b(1)\ \ 0 \ldots 0\ b(q-1)\ b(q)]$.

Step4.continue to find all parameters

Similar to steps 2 and3, in each step obtain b(k), b(q-k).

Note1: The convergence of proposed algorithm is based on equations (6-9) that considered in previous section.

Note2: As equation (5) is a criterion based on cumulants and these are delay insensitive it can be concluded that the estimated parameters may have a phase delay in comparison with the original ones. Therefore, a delay mismatch between the estimated and original parameters may occur.

Note3: In each step of the genetic algorithm, x(1) and x(2) are found. In our experimental results, we obtain four parameters in each step using the setting given below for the MATLAB GA algorithm. It means in each step, b(k), b(q-k), b(k+1), b(q-

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k-1) are obtained using the above mentioned steps 2-4 . The computation time is reduced doing so.

Note4: The accuracy of the proposed algorithm can be proved by considering the differences between 1-D diagonal slice of output cumulants and cumulants that are obtained by inserting the estimated parameters in equation (4). The results (section 6) show the accuracy and convergence of the proposed algorithm.

Note5: This algorithm can be repeated after finding all parameters of sequence b , and setting this sequence as initial values for next running the algorithm from step 2.

GA Operators:

All GAs are effective when used with its best operations and values of parameters. The following operators are modified due to experimental results.

1) The fitness function is considered in equation (5) that must be optimized. For standard optimization algorithm this is known as the objective function.

2) The population size determines the size of the population at each generation. Choosing the population size as 50 is satisfied the results.

This optimization includes two variables (four variables in our case) so the population can be represented with a 50 by 2 matrix, at each iteration, the genetic algorithm performs a series of computational on the current population to produce a new population. The algorithm begins by creating a random initial population in the interval of [-1, 2.5].

3) At each step, the genetic algorithm uses the current population to create the children that make up the next generation. Algorithm usually selects individuals that have better fitness values. "Stochastic uniform" used as "selection" mechanism, it is robust and simple.

4) Elite children are the number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. "Elite count" considered in this paper is 10 or 20% of population size.

The "termination criterion" is reaching at 200th generation that means algorithm is repeated until the number of generations equal to 200.

5) "Gaussian mutation" and "heuristic crossover" used to produce offspring for the next generation. As known a Gaussian mutation operator requires two parameters: the mean, which is often set to zero, and the standard deviation δ . δ =5.00 in our case.

Suppose a child is considered as a value will be produced in the next generation and parents are the values which obtained in previous generation, "heuristic crossover" returns a child that lies on the line containing the two parents, a small distance away from the parent with the better fitness value in the direction away from the parent with the worse fitness value.

The parameter Ratio can specify how far the child is from the better parent. The following equation illustrates the relation between parameter Ratio and child (as next generation).

Child= parent 2 + R (parent 1- parent 2)

Where parent 1 & parent 2 are the parents of the current generation, and of course parent 1 has a better value, and R is the "parameter Ratio". R=1.2 is considered in this work.

6) Using "Hybrid function" increases the robustness of genetic algorithm. A hybrid function is an optimization function that runs after the genetic algorithm terminates in order to improve the value of fitness function. The Hybrid function uses the final point from the genetic algorithm as its initial point. We use the function "fminsearch" an un constrained minimization function in the optimization toolbox of MATLAB. "fminsearch" uses the simplex search method of [18]. This is a direct search method that does not use numerical or analytic gradients.

VI. SIMULATION RESULTS

In the following examples, we use our recursive algorithm to identify MA model from output data only. Also for the input sequence, independent exponentially distributed random deviates ($\delta^2_{\exp} = 1, \gamma_{\exp} = \gamma_3 = 2$) are generated by using HOSA Toolbox [19]. To find the estimate $\hat{c}_3$ (third order cumulant) needed for our MA identification algorithm, we used N output samples which are computed by convolving the random input with the true MA model. The N (640 or 1024 or 2048) output samples were divided into (5 or 8 or 16) records, respectively, each containing 128 samples.

$$\hat{c}^{(i)}(m) = \frac{1}{128} \sum_{k=\max(0,-m)}^{\min(128,128-m)} y^{(i)}(k)\left[y^{(i)}(k+m)\right]^2,$$

$$i = 1, 2, \cdots, M;$$

$$m = -q, \cdots, 0, \cdots, q.$$

To reduce the variance of the $\hat{c}_3$ estimator, we averaged over the M-records to obtain

$$\hat{c}(m) = \frac{1}{M} \sum_{i=1}^{M} \hat{c}^{(i)}(m)$$

Also Gaussian noise was added to produce signal to noise ratio (SNR) level of 10dB as in [2].

Example 1: (NMP-MA with q=12) the true non-minimum phase model is (according to figure (2))

$$\begin{aligned} x(i) &= 10w(i) + 20w(i-1) + 31w(i-2) - 40w(i-3) \\ &\quad - 23w(i-4) + 19w(i-5) - 50w(i-6) - 11w(i-7) \\ &\quad + 12(i-8) + 39(i-9) + 43(i-10) - 30(i-11) \end{aligned}$$

Figure (6) shows the estimated parameters versus original parameters after running our recursive algorithm only once, also figure (7) shows the 1-D diagonal slice of third order cumulant of output data and 1-D diagonal slice of third order cumulant obtained by inserting our estimated parameters in equation (4). Moreover, it illustrates the accuracy of the algorithm. In accordance it can be concluded that, although the

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estimated parameters do not match the originals exactly, but the used criterion is satisfied correctly.

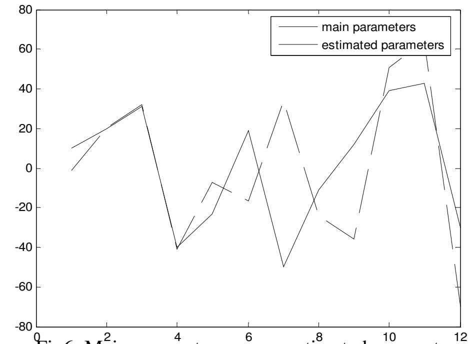

Fig6: Main parameters versus estimated parameters

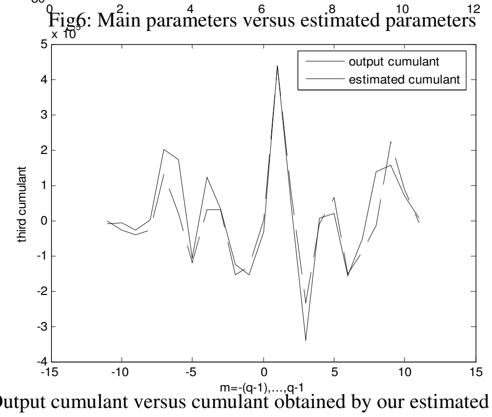

Fig7: Output cumulant versus cumulant obtained by our estimated parameters inserted in equation (4)

Example 2: (NMP-MA with q=16) the true non-minimum phase model is (according to figure (2))

$$\begin{aligned} x(i) &= 10w(i)+20w(i-1)+31w(i-2)-40w(i-3) \\ &-22w(i-4)+31w(i-5)+10w(i-6)-42w(i-7) \\ &+31w(i-8)+12w(i-9)-30w(i-10)+14w(i-11) \\ &-23w(i-12)+56w(i-13)+9w(i-14)-2w(i-15) \end{aligned}$$

Figure (8) shows the estimated parameters versus main parameters after running our recursive algorithm only once and figure (9) shows the 1-D diagonal slices of the third order cumulant of output data and that obtained by our estimated parameters. Moreover the comparison between our estimated and the main parameters, show that our estimated parameters have a phase delay in comparison with main parameters that comes from statistical criteria.

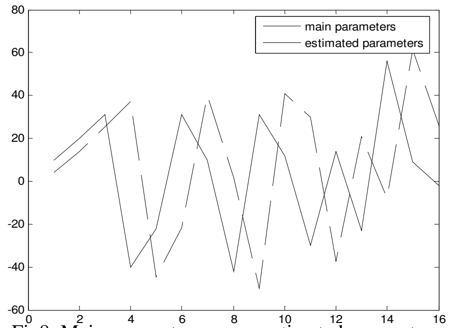

Fig8: Main parameters versus estimated parameters

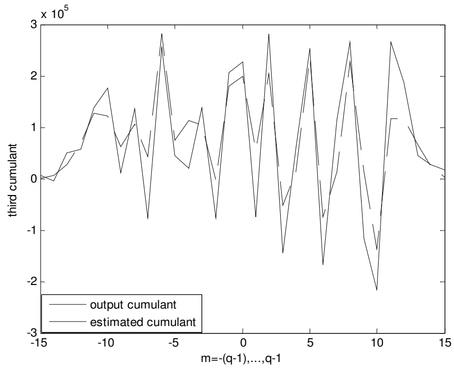

Fig9: Output cumulant versus cumulant obtained by estimated parameters inserted in equation (4)

Example 3: (NMP-MA with q=20) the true non-minimum phase model is (according to figure (2))

$$\begin{aligned} x(i) &= 10w(i)+20w(i-1)+31w(i-2)-40w(i-3) \\ &-22w(i-4)+31w(i-5)+10w(i-6)-42w(i-7) \\ &+31w(i-8)+12w(i-9)-30w(i-10)+14w(i-11) \\ &-23w(i-12)+56w(i-13)+9w(i-14)-2w(i-15) \\ &+12w(i-16)-13w(i-17)+10w(i-19) \end{aligned}$$

Figure (10) shows the estimated parameters using our recursive algorithm, versus the original parameters after running our recursive algorithm for three times, also figure (11) shows the 1-D diagonal slices of the third order cumulant of output data and that obtained by our estimated parameters.

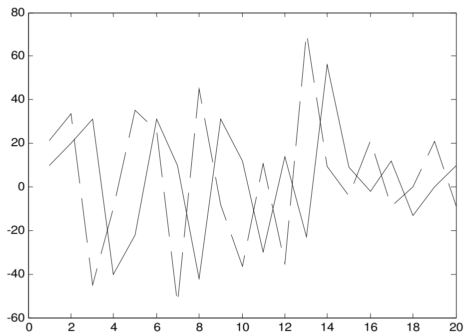

Fig10: Main parameters versus estimated parameter

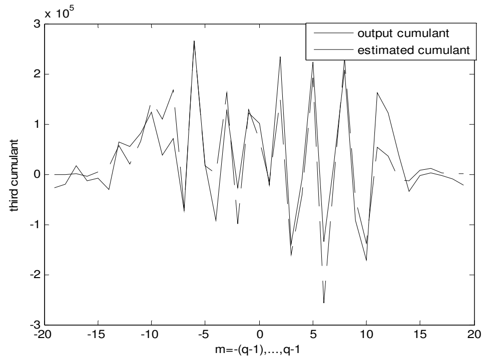

Fig11: Output cumulant versus cumulant obtained by our estimated parameters inserted in equation (4)

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VII. CONCLUSION

A novel method for blind identification the MA parameters of a non-minimum phase system, have been proposed in this paper. We have used one-dimensional versions of third order cumulants for parameter estimation and presented both analytical and optimization-based solutions for parameter estimation, also we have shown that un-known parameters can be identified step by step. Unlike other researches, our estimated parameters are not sensitive to the filter order. We have used Genetic Algorithm optimization as optimization algorithm, in order to update unknown parameters in each step. Moreover the results show the accuracy of proposed method and the ability of our algorithm in identifying the parameters of filter where the order of MA model exceeds four.

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