BLIND IDENTIFICATION OF NON-MINIMUM PHASE FIR SYSTEMS USING HIGHER ORDER STATISTICS AND HYBRID GENETIC ALGORITHM

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ABSTRACT

In this paper we present a novel method for identification of linear time invariant, non-minimum phase (NMP), FIR systems when only output data are available and the order of system exceeds four. We generally model a non-minimum phase FIR system as an MA model of known order. To estimate the model parameters, we exploit the 1-D diagonal slice of the third order cumulant of the output which may be contaminated by additive, zero mean, Gaussian white noise of unknown variance. This method is based on a new viewpoint of the third order cumulant equation and uses a recursive optimization method. The unknown parameters are estimated satisfying the minimization of the sum of squared differences between the observed cumulant (diagonal slice of output) and the cumulant of the proposed model. We propose both analytical and optimization-based solutions for identifying the parameters. The analytical solution serves as a basis for applying our recursive optimization method to obtain parameters step by step. A hybrid algorithm is used where genetic algorithm provides initial values for Nelder-Mead Simplex Optimization Method. Moreover, experimental results indicate the robustness and accuracy of our proposed algorithm for high order FIR filters.

***Index Terms*—** Blind System Identification, Hybrid Genetic Algorithm, Higher Order Statistics

1. INTRODUCTION

During recent years higher order statistics (HOS) have begun to find wide applicability in many diverse fields e.g., sonar, radar, plasma physics, biomedicine, seismic data processing, image reconstruction, harmonic retrieval, time delay estimation, adaptive filtering, array processing, blind deconvolution (equalization) and blind system identification[1]. These statistics known as cumulants and their associated Fourier transforms, known as poly spectra, not only reveal amplitude information about a process but, also reveal phase information. This is important because, as is well known, second order statistics are phase-blind. The HOS-based blind deconvolution and system identification, associated with nonlinear filtering, are classified as explicit and implicit solutions [2]. The implicit solutions include the well known Bussgang algorithm. In fact, implicit HOS algorithms, such as our proposed method, are relatively simple to implement and are generally capable of delivering a good performance, as evidenced by their use in digital communication systems. However, they suffer from two basic limitations: a potential convergence to a local minimum and sensitivity to phase jitter. In contrast, explicit HOS-based solutions, being closed form, overcome the local minimum problem by avoiding the need for minimizing a cost function; unfortunately, they are computationally much more complex. Both explicit and implicit kinds of HOS-based solutions suffer from slow rate of convergence due to the fact that the time-average estimation of higher order statistics requires a large sample size. Furthermore, the speed of convergence and accuracy of these solutions are at the detriment of the system order and worsen with increasing q (filter order). Nevertheless, various recursive and least squares methods for the identification of MA systems have been proposed using a variety of second, third and fourth order statistics and different 1-D cumulant slices [3], [4], [5-7]. Mendel in [1] categorized the methods for blind MA system identification in three groups: 1) closed-form solutions, 2) linear algebra solutions, 3) optimization solution. In [8] a new method for MA parameter estimation which exploits all samples of the second and third order cumulants to reconstruct the unknown system impulse response is developed. References [9] and [10] proposed methods that depend on third order cumulants alone. Method [11] is remarkably different as it uses a combination of second, third and fourth order statistics. This aim in using this combination of cumulant orders is to support intrinsically and automatically all non-Gaussian input distributions. Hence method [11] can be used more widely and with more confidence regardless of prior knowledge of system statistics. For an MA process the system output is related to the input by the convolution sum of the input with the rational system transfer function $B(z)$ in figure (1).

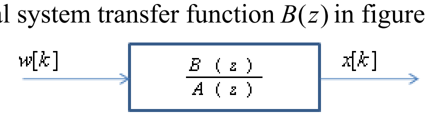


Fig1: Model Transfer Function

Generally speaking two techniques are employed in the estimation of the filter coefficients: nonlinear methods and linear simplifications [3]. Nonlinear solutions can be computationally expensive and may converge to local minimum [8]. However, when these nonlinear methods are properly initialized, the estimates obtained are usually better than the estimates obtained using linear methods. MN96 is a nonlinear method which uses the $c(q,k)$ formula to initialize the algorithm [3]. These mentioned methods assume prior knowledge of MA model order, q. In fact, estimation of q from the time series is a substantial part of the system identification problem. Hence, model order selection has become an area of research in its own right.

Up to now, researches in ([2], [3], [4], [6], [10], [15]) have concentrated mostly on identifying MA filter coefficients with the order lower than five. The main problem of HOS-based MA parameter estimation for high order filters is due to propagation errors in estimating the parameters which is derived from the high nonlinearity of cumulant-based equations. This problem is solved by the proposed method in this paper whose advantage in obtaining the system parameters higher than 4 derives from its recursive characteristic.

Organization of this paper is as follows: in section 2, definitions of MA filters and third order cumulant equation are defined. The problem of MA parameter estimation using third order cumulants have been stated in section 3, and in section 4, a new viewpoint on obtaining MA filter coefficients is shown and the proposed algorithm is explained in section 5. And, finally in section 6, the simulation results are shown to indicate the accuracy and robustness of the proposed algorithm.

2. MA FILTERS AND HIGHER ORDER STATISTICS

Following [3], consider the system depicted schematically in figure (2). The noise-free signal, $x[k]$, is related to the driving noise, $w[k]$, by parameters $b[i]$ where i takes the values 0, $\cdots$, q and q is the model order. The observed output, $y[k]$ is corrupted by additive Gaussian noise, $v[k]$.

Figure (3) shows a more generic representation of the MA process where the filtering operation is depicted by the function *B(z)*. For an MA process, *B(z)* is a rational polynomial with order q equal to the number of previous inputs that affect the current output. The parameter q gives the order of the process. The noise-free output, $x[k]$, is related to the input, $w[k]$, by constant weights, $\{b(i)\}_{i=0}^{q}$, given in equation (1). The corruption of this output by noise results in equation (2) where the observed noise-corrupted signal, $y[k]$, is the summation of the noise-free signal and the additive noise $v[k]$.

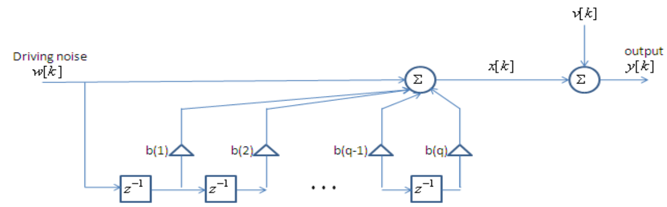


Fig2: MA Modeling

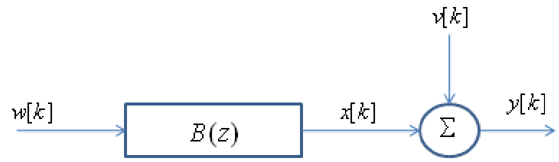


Fig3: Notation for Cumulant Identification Methods

$$x[k]=\sum_{i=0}^{q} b(i)w[k-i] \tag{1}$$

$$y[k]=x[k]+v[k] \tag{2}$$

If higher order statistics are to be used to formulate a general relationship for the identification of the model parameters from the output of the system only, then the following conditions 1 and 2 must hold:

1. The driving noise, $w[k]$, is zero-mean, independent and identically distributed (i.i.d) and non-Gaussian with $E\{w^2[k]\}=\gamma_{2w}$,
2. $E\{w^3[k]\}=\gamma_{3w}\neq 0$ and $E\{w^4[k]\}-3\gamma_{2w}=\gamma_{4w}\neq 0$
3. The measurement noise, $v[k]$, is assumed to be zero-mean, i.i.d., and independent of $w[k]$. In addition it is assumed to be Gaussian in distribution with $E\{v^2[k]\}=\sigma_v^2$, $E\{v^3[k]\}=\gamma_{3v}=0$, $E\{v^4[k]\}-3\sigma_v^2=\gamma_{4v}=0$.

Let $c_{3x}(m,n)$ represent the third order cumulant of the observed noise-free signal at lags m and n. Equation (3) relates the third-order cumulant at the specified combination of lags to the MA parameters $\{b(i)\}_{i=0}^{q}$ and the skewness of the input to the system, γ_{3w} [1],[15].

$$c_{3x}(m,n)=\gamma_{3w}\sum_{k=0}^{q} b(k)b(k+m)b(k+n) \tag{3}$$

This relation is the basis for all methods of blind system identification which make use of third-order statistics.

3. PROBLEM STATEMENT

Equation (3) can be modified to yield an equation using the diagonal slice of the third order cumulant. This method is obtained by setting $m=n$ in equation (3) generating a relation between the diagonal cumulant slice $c_{3x}(k,k)$ and the MA parameters to be estimated. Using the effect of HOS in omitting Gaussian noise [1], [3], we have:

$$c_{3y}(m,m) = c_{3x}(m,m)$$

$$c_{3y}(m,m) = \gamma_{3w} \sum_{k=0}^{q} b(k)b(k+m)^2 \qquad m=-q,\ldots,0,\ldots,q \tag{4}$$

The fitness function which should be minimized according to equation (4) is as follow:

$$J = \sum_{m=-q}^{q} [c_{3y}(m,m) - \gamma_{3w} \sum_{k=0}^{q} b(k)b(k+m)^2]^2 \tag{5}$$

In [12] two methods for the estimation of the MA coefficients based on the $c_{3x}(k,k)$ formula are suggested: a nonlinear least-square solution and a linear programming approach. The computational overhead in these methods is very high and rapidly increases with the model order; and similar to other previous works these methods are only appropriate for low order filters ($q \le 4$).

We present a novel viewpoint in equation (3) that allows us obtain unknown parameters recursively. We will show next, the analytical basis of our optimization method to estimate the parameters of a non-minimum phase MA filter.

4. ANALYTICAL VIEWPOINT

According to (3), the following equations can be obtained by simple mathematical manipulations:

$$\begin{cases} c_{3y}(q,q) = \gamma_{3w} b(0)b(q)^2 \\ c_{3y}(-q,-q) = \gamma_{3w} b(0)^2 b(q) \end{cases} \tag{6}$$

$$\begin{cases} c_{3y}(q-1,q-1) = \gamma_{3w}[b(0)b(q-1)^2 + b(1)b(q)^2] \\ c_{3y}(-(q-1),-(q-1)) = \gamma_{3w}[b(0)^2 b(q-1) + b(1)^2 b(q)] \end{cases} \tag{7}$$

$$\begin{cases} c_{3y}(q-2,q-2) = \gamma_{3w}[b(0)b(q-2)^2 + b(1)b(q-1)^2 \\ \qquad\qquad + b(2)b(q)^2] \\ c_{3y}(-(q-2),-(q-2)) = \gamma_{3w}[b(0)^2 b(q-2) + b(1)^2 b(q-1) \\ \qquad\qquad + b(2)^2 b(q)] \end{cases} \tag{8}$$

$$\vdots$$

$$\vdots$$

$$c_{3y}(0,0) = \gamma_{3w}(b(0)^3 + b(1)^3 + \cdots\cdots + b(q)^3) \tag{9}$$

It can be seen from the above system of equations that: $b(0)$, $b(q)$ can be obtained by solving Eq(6). Parameters $b(1)$, $b(q-1)$ are then obtained using $b(0)$, $b(q)$ and solving Eq(7). Coefficients $b(2)$, $b(q-2)$ are estimated using $b(0)$, $b(q)$ and $b(1)$, $b(q-1)$ and solving Eq(8) by the same token. Other parameters are defined in the same way recursively.

5. OPTIMIZATION BASED SOLUTION

In this part, we describe our algorithm which is based on a recursive estimation of unknown parameters. Consider the following model.

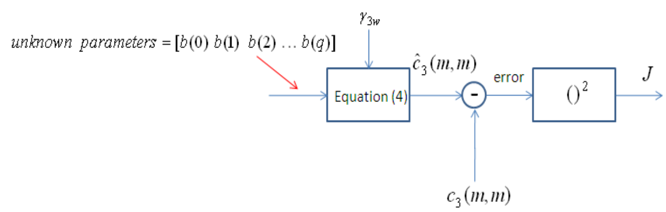


Fig3: Model of the proposed algorithm

Where γ_{3w} is the skewness of the input signal and is simply calculated using the method explained in [15]. $c(m,m)$ is the 1-D diagonal slice of third order cumulant of the output, and, $\hat{c}(m,m)$ is calculated using equation (4). The sequence of the unknown parameters b must be determined by minimizing equation (5). We propose the following algorithm based on the recursive dependence of parameters to each other as seen in equations (6-9).

5.1. Proposed Hybrid Algorithm

A- Genetic algorithm

The genetic algorithm whose output provides the initiation for the Nelder-Mead Simplex Optimization Method is described in the steps below:

Step1.Initial value:

Set b=zeros(1,q+1), where q is the order of filter (in this paper, it is assumed that the order of filter is definite).

Step2.Finding b(0) and b(q)

Insert x(1)=b(0) and x(2)=b(q), and find x(1) and x(2) by minimizing equation (5), using genetic algorithm optimization. Consider to figure (4) which shows the model of finding b(0) and b(q).

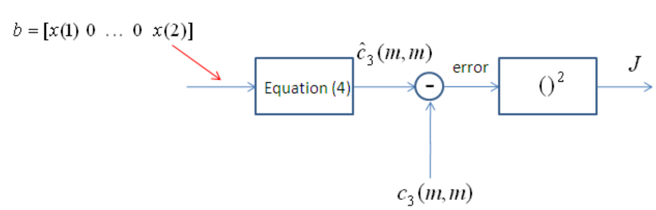


Fig4: Model of second step of proposed algorithm

After this step, b(0)=x(1), b(q)=x(2). The sequence of MA filter coefficients (b) will be $b = [b(0)\ 0 \ldots 0\ b(q)]$.

Step3.Finding b(1) and b(q-1)
Using b(0) and b(q) from step 2 and insert x(1)=b(1) and x(2)=b(q-1), then find x(1) and x(2) by minimizing equation (5), using genetic algorithm optimization. Consider to figure (5) which shows the model of finding b(1) and b(q-1).

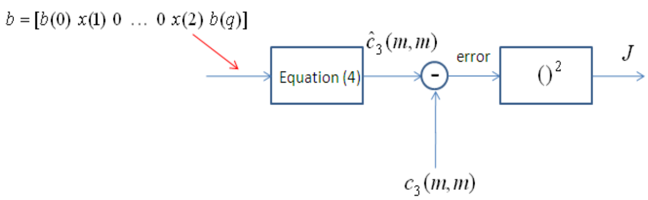


Fig5: Model of third step of proposed algorithm

After this step, b(1)=x(1), and b(q-1)=x(2). The sequence of MA filter coefficients (sequence b) will be as follow: $b=[b(0)\ b(1)\ \ 0\dots0\ b(q-1)\ b(q)]$.

Step4.Continue to find all parameters
Similar to steps 2 and3, in each step obtain b(k), b(q-k).

B- Nelder-Mead Simplex Optimization Method Function
We use "fminsearch" function of MATLAB as our main optimization tool. Note that in this step all parameters (filter coefficients) are obtained simultaneously. Function "fminsearch" uses the simplex search method of [13]. This is a direct search method that does not use numerical or analytic gradients. If n is the length of x, a simplex in n-dimensional space is characterized by the n+1 distinct vectors that are its vertices. In two-space, a simplex is a triangle; in three-space, it is a pyramid. At each step of the search, a new point in or near the current simplex is generated. The function value at the new point is compared with the function's values at the vertices of the simplex and, usually, one of the vertices is replaced by the new point, giving a new simplex. This step is repeated until the diameter of the simplex is less than the specified tolerance.

Note1: A large number of simulations have approved the convergence of our algorithm but two matters should be noted in this way; first, the direction of estimated parameters which is based on the analytical viewpoint (equations 6-9) and, Second, at each step, the complete length of generated $\hat{c}(m,m)$ is compared with $c(m,m)$ as the optimization criterion.

Note2: As equation (5) is a criterion based on cumulants that are delay insensitive it can be concluded that the estimated parameters may have a phase delay in comparison with the original ones. Therefore, a delay mismatch between the estimated and original parameters may occur.

Note3: The accuracy of the proposed algorithm can be demonstrated by considering the differences between 1-D diagonal slice of output cumulants and cumulants that are obtained by inserting the estimated parameters in equation (4). The results (section 6) show the accuracy and convergence of the proposed algorithm.

Note4: In each step of the genetic algorithm, x(1) and x(2) are found. In our experimental results, we obtain four parameters in each step using the setting given below for the MATLAB GA algorithm. It means in each step, b(k), b(q-k), b(k+1), b(q-k-1) are obtained using the above mentioned steps 2-4 . The computation time is reduced doing so.

Note5: The hybrid algorithm can be repeated once all filter parameters are calculated by using the calculated sequence as initial values and running the above algorithm from step 2 a second time.

5.2. Genetic Algorithm Parameters

A genetic algorithm is most effective when its parameters are set properly. The following setting is used in our experiments.
1) The equation (5) is considered as fitness function that must be optimized. This is also known as objective function.
2) The population size determines the size of the population at each generation. Choosing the population size 20 is satisfactory for our results.
The optimization involves two variables (four variables in our case) so the population is represented with a 20 by 2 matrix. The genetic algorithm performs, at all iterations, a series of computations on the current population to produce a new one. The algorithm begins by creating a random initial population in the interval [-1, 2.5] as initial range.
3) In each step, the genetic algorithm uses the current population to create the children that make up the next generation. The algorithm usually selects individuals that have better fitness value.
The selection mechanism is "stochastic uniform". It is robust and simple.
4) Elite children are the number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. Elite count considered in this paper is 5 or 25% of the population size.
The termination criterion is reaching 200^{th} generation that means the algorithm is repeated until the number of generations equals 200.
5) "Gaussian mutation" and "heuristic crossover" are used to produce offspring. A Gaussian mutation operator requires two parameters: the mean, which is often set to zero, and the standard deviation δ . δ =6.732 is used here.
Suppose a child is considered as a value that will be produced in the next generation and parents are the values which are obtained in the previous generation. "Heuristic crossover" returns a child that lies on the line containing the

two parents, a small distance away from the parent with the better fitness value in the direction away from the parent with the worse fitness value.
The parameter "Ratio" specifies how far the child is distant from the better parent. The following equation illustrates the relation between parameter Ratio and child (as next generation).

Child= parent 2 + R (parent 1- parent 2)

Where parent1 & parent2 are the corresponding parents, and of course parent1 has the better fitness value and R is the parameter Ratio. R=1.2 is used here.

6. SIMULATION RESULTS

In the following examples we use our recursive algorithm to identify an MA model from output data only. For the input sequence, independent exponentially distributed random deviates ($\delta_{\exp}^2 = 1, \gamma_{\exp} = \gamma_3 = 2$) are generated by using HOSA Toolbox [14]. To find the estimate third order cumulant $\hat{c}_3$ needed for our MA identification algorithm we use N output samples which are computed by convolving the random input with the true MA filter. The N (640 or 1024 or 2048) output samples were divided into (5, 8 or 16) records, respectively, each containing 128 samples.

$$\hat{c}^{(i)}(m) = \frac{1}{128} \sum_{k=\max(0,-m)}^{\min(128,128-m)} y^{(i)}(k)\left[y^{(i)}(k+m)\right]^2,$$
$$i = 1, 2, \cdots, M;$$
$$m = -q, \cdots, 0, \cdots, q.$$

To reduce the variance of the $\hat{c}_3$ estimator we average over the M-records to obtain:

$$\hat{c}(m) = \frac{1}{M} \sum_{i=1}^{M} \hat{c}^{(i)}(m)$$

Also Gaussian noise was added to produce signal to noise ratio (SNR) level of 10dB as in [15].

Example 1: (NMP-MA with $q=8$) the original non-minimum phase filter is (according to figure (2))

$$x(i) = 10w(i) + 20w(i-1) + 31w(i-2) - 40w(i-3)$$
$$-22w(i-4) + 31w(i-5) - 10w(i-6) - 42w(i-7)$$

Figure (6) shows the estimated versus original parameters after running our recursive hybrid algorithm only once. Figure (7) compares the third order cumulant of the output data and third order cumulant obtained by our estimated parameters (equation (4)).
Moreover, it illustrates the accuracy of the algorithm. In accordance it can be concluded that, although the estimated parameters do not match the originals exactly, but the used criterion is satisfied correctly.

Example 2: (NMP-MA with q=16) the original non-minimum phase filter is:

$$x(i) = 10w(i) + 20w(i-1) + 31w(i-2) - 40w(i-3)$$
$$-22w(i-4) + 31w(i-5) + 10w(i-6) - 42w(i-7)$$
$$+31w(i-8) + 12w(i-9) - 30w(i-10) + 14w(i-11)$$
$$-23w(i-12) + 56w(i-13) + 9w(i-14) - 2w(i-15)$$

Figure (8.a) shows the estimated versus original parameters after running our recursive hybrid algorithm only once. Figure (8.b) gives the third order cumulant of the output data and that obtained by our estimated parameters. Moreover, figure (8.c) shows the estimated versus original parameters after running our recursive algorithm twice. Also, figure (8.d) gives the third order cumulant of the output data and that obtained by our estimated parameters. The latter mentioned figures demonstrate the effect of repeating the proposed algorithm on the precision of estimated parameters.

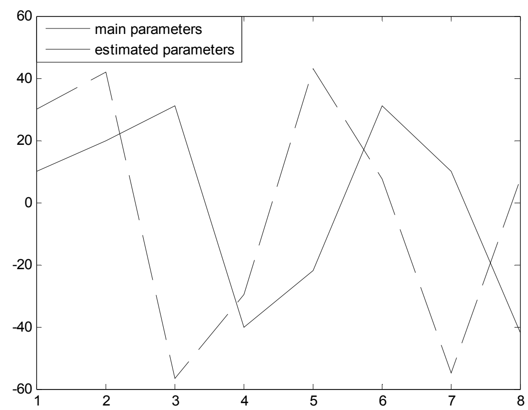


Fig6: Estimated versus original parameters

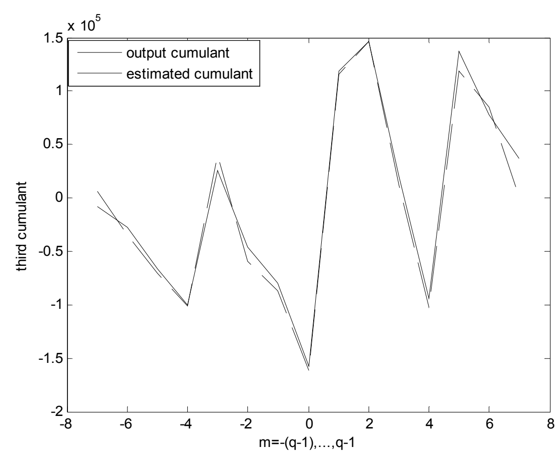
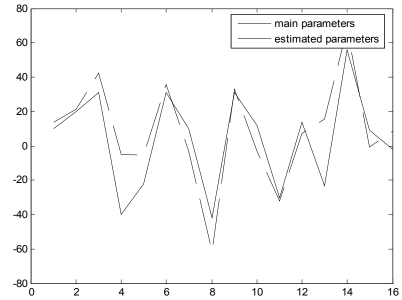
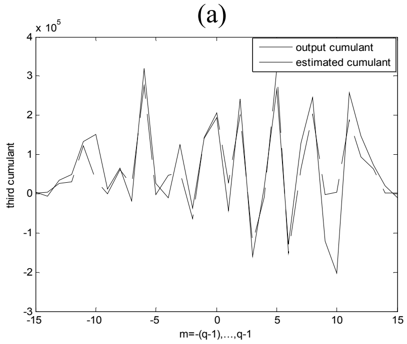


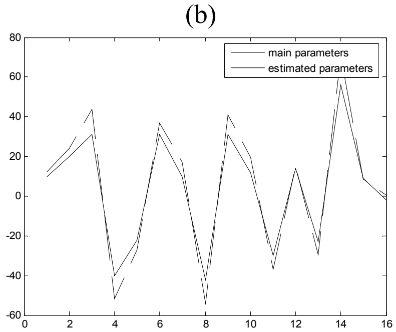
Fig7: Output cumulant versus cumulant obtained by our estimated parameters used in equation (4)



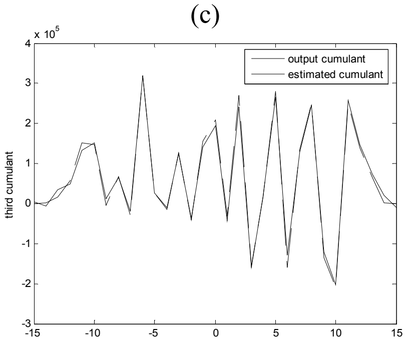
(a)



(b)



(c)



(d)

Fig 8. (a). Estimated versus original parameters after running algorithm once, (b). output cumulant versus cumulant obtained by our estimated parameters used equation (4), (c) Estimated versus original parameters after running algorithm twice and (d). output cumulant versus cumulant obtained by our estimated parameters inserted in equation (4).

8. CONCLUSION

A novel method for blind identification of a non-minimum phase MA system has been proposed. We have shown that the unknown parameters can be identified using a hybrid genetic algorithm and, unlike other methods, our estimated parameters are not sensitive to the system order (number of filter coefficients). Moreover, the results have shown the accuracy of the proposed method and its ability in identifying the parameters recursively where the order of MA model is higher than four. It is noted, however, that the key issue in this method, as in other parametric approaches, is the selection of the model order and the sensitivity of the method to model mismatch. Actually, the cumulant lags used in the optimization depend directly on the model order. These issues are not considered in this work and form our future line of studies.

12. REFERENCES

[1] Jerry M. Mendel, "*Tutorial on Higher Order Statistics in Signal Processing and System Theory*" PROCEEDINGS of IEEE, VOL.79, NO.3, MARCH 1991.

[2] Haykin, S.; "Adaptive filter theory", Prentice Hall, 2001.

[3] JonesA McCormick, A.K. Nandi "*Higher Order and Cyclostationary Statistics*".

[4] J.A.Fonollosa and J.Vidal "*System Identification using a Linear Combination of Cumulant Slices*" PROCEEDINGS of IEEE, 41:2405-2411, 1993.

[5] G.B.Giannakis "*Cumulants: A Powerful Tool in Signal Processing*" PROCEEDINGS of IEEE, 75:1333-1334, 1989.

[6] A.K.Nandi and R.Mehlan, "*Parameter Estimation and Phase Reconstruction of Moving Average Processes using Third Order Cumulants*" Mechanical Systems and Signal Processing, 8:421-436, 1994

[7] J.K.Tugnait "*New Results on FIR system Identification using Higher Order Statistics*" IEEE TRANSACTION on Signal Processing, 39:2216-2221, 1991.

[8] S.A.Alshebeili, A.N.Venetsanopoulos, and A.E.Cetin, "*Cumulant Based Identification Approaches for Non- Minimum Phase FIR Systems*" PROCEEDINGS of IEEE, 41:1576-1588, 1993.

[9] A.K.Nandi "*Blind Identification of FIR Systems using Higher Order Cumulants*" Signal Processing, 39:131-147, 1994.

[10] X.Ahang and Y.Zhang "*FIR System Identification using Higher Order Statistics Alone*" IEEE INSTRUMENTATION on Signal Processing, 24:2854-2858, 1994.

[11] J.K.Martin and A.K.Nandi "*Blind System Identification using Second, Third and Fourth Order Cumulants*" Journal of Franklin Institute of Science, 333B:1-13, 1996.

[12] K.S.Lii and M.Rosenblatt, "*Deconvolution and Estimation of Transfer Function Phase and Coefficients for Non-Gaussian Linear Process.*" Annals of Statistics, 10:1195-1208, 1982.

[13] Lagarias, J.C., J. A. Reeds, M. H. Wright, and P. E. Wright, "*Convergence Properties of the Nelder-Mead Simplex Method in Low Dimensions* " SIAM Journal of Optimization, vol. 9 Number 1, pp. 112147, 1998.

[14] Ananthram Swami, Jerry M.Mendel, Chrysostomos L.Nikias "*Higher Order Spectral Analysis Toolbox for Use with MATLAB*" January 1998 third printing (MathWorks).

[15] G.B.Giannakis and J.M.Mendel "*Identification of non-minimum Phase Systems using Higher Order Statistics*" IEEE INSTRUMENTATION on Acoustics, Speech and Signal Processing, 37:360-377, 1989.