# Parameter Estimation Optimization Based on Genetic Algorithm Applied to DC Motor

M. Lankarany and A. Rezazade

Department of Electrical and Computer Engineering, Shahid Beheshti University of IRAN

## 1. mlankar@yahoo.com,

2. alireza rezazade@yahoo.com

Abstract-This paper proposed the application of Genetic Algorithm Optimization in estimating the parameters of dynamic state of DC Motor. LSE estimation is considered as a convenient method for parameter estimation, in comparison with this proposed method. Despite of LSE estimation that is based on the linearity of error function due to parameters, GA method can easily identify unknown parameters by minimizing the sum of squared errors. GA is imported in comparison with conventional optimization methods because of its power in searching entire solution space with more probability of finding the global optimum. Also the model can be nonlinear with respect to parameters, and in this identification free noise system is assumed and transient excitation is considered instead of persistent excitation. Finally comparison between LSE and GA optimization is presented to indicate robustness and resolution of GA identification method in parameter estimation.

#### I. INTRODUCTION

The physically reality that provides the experimental data will generally be referred to as the process. In order to perform a theoretical analysis of identification, it is necessary to impose assumptions on the data. In such cases the word system will be used to denote a mathematical description of the process. In practice, where real data are used, the system is unknown and can even be an idealization. For simulated data, however, it is not only known but also used directly for the data generation in the computer. In this paper the dynamic treatment of DC Motor is considered as the system that must be identified with proposed parameter estimation method [1] The system identification includes typically two models, 1-non parametric models and 2- parametric models. Non parametric models correspond to such models which described by a curve, function or table. However in many cases, it is relevant to deal with parametric models. Such models are characterized by a parametric vector which will be denoted by  $\theta$ . When  $\theta$ is varied over some set of feasible values we will obtain the structure of dynamic model of DC Motor.

[1] In general terms, the experimental condition is a description of how, the identification experiment is carried

out. This includes the selection and generation of the input signal, the sampling interval, pre filtering of data prior to estimation of the parameters, etc. before turning to parameter estimation of dynamic model of DC Motor, it should be mentioned that the model of system must be regarded as fixed. It means the experimental condition is determined when the data are collected from the process and since collecting data the characteristic property of system can not be changed by the user. LSE estimation, that is considered as compared with Genetic Algorithm Estimation, is a kind of on line parametric method when it is used recursively. After introducing LSE method and using this method for identifying the dynamic state of DC Motor, the genetic algorithm optimization will be used and the results of both methods will be compared together.

In this paper, we will introduce the application of genetic algorithm optimization in parametric method identification. Unknown parameters will be obtained since minimizing the error function is completed. In order to treat the parameter estimation problem by the GA, a fitness function must first be formulated, according to the sum squared error. The proposed algorithm begins with a collection of parameter estimates (chromosomes), and each one is evaluated for its fitness in solving the given optimization task. In each generation, the chromosomes with higher fitness values are allowed to mate and bear offspring. These children (new parameter estimates) then form the basis for the next generation because of use of crossover and mutation, this parameter estimation algorithm tends to find the global optimum solution without being trapped at local minimum. GA has been successfully applied to a variety of optimization problems, such as image processing, system identification, and fuzzy logic controller design [1,2,3].

#### II. About This Method

This proposed method is based on following stages: *A*. Assume **G** is a LTI system which it's specific model is undefined (only we can guess the type and number of poles) so H(s) (the estimated transfer function of system) can be expressed as a ratio of polynomials in s(Laplace) domain as follow:

$$H(s) = \frac{\prod_{k=1}^{M} (s - z_k)}{\prod_{k=1}^{N} (s - p_k)} \qquad N \ge M \tag{1}$$

*B*. Assume the system includes N1 real poles and  $2 \times N2$  complex poles. So the partial fraction expansion of H(s) can be obtained as follow:

$$H(s) = \sum_{k=0}^{N_1} H_{1k}(s) + \sum_{k=0}^{N_2} H_{2k}(s)$$
<sup>(2)</sup>

Where 
$$H_{1k}(s) = \frac{a_{1k}}{s + b_{1k}s}$$
 (3)

And 
$$H_{2k}(s) = \frac{a_{2k}s + a'_{2k}}{s^2 + b_{2k}s + b'_{2k}}$$
 (4)

Where  $a_{1k}$ ,  $a_{2k}$ ,  $b_{1k}$ ,... are unknown parameters.

*C*. assume the system is excited with a step function as an input of following model:



Fig. 1. Assumed model

$$Y(s) = X(s)H(s)$$
 &  $H(S) = \int_{0}^{\infty} e^{-st}h(t)dt$  (5)

As a result the step response of the system is obtained as equation (6)

$$y(t) = \int_{0}^{t} h(v) dv \& h(t) = \{H(s)\}^{-1}$$
(6)

y(t) can be easily calculated in general form "(2),(3),(4)," where inverse Laplace transform of H(s) is available in any basic mathematics books or in MATLAB symbolic mathematic)

*D*. after calculating y(t) corresponds to unknown parameters( equations (2,3,4)), the following model is applicable.



Fig. 2. The model of estimation

 $0 \le i \le N$ , G is unavailable system (only out put data points are available) and y(t) is estimated function corresponds to unknown parameters.

Now consider 
$$\sum e^2 = \sum_{k=0}^{N} (g(kT_s) - y(kT_s))^2$$
 where in each

sample time,  $g(kT_s)$  is real number and  $y(kT_s)$  is a function of unknown parameters in that sample time.( h(t) can be obtained with any information about the location of poles or guessing the dynamic state of model and we don't have h(t) or H(s) according to specific modeling, so if the specific model is available the problem will be too easy.)

*E.* in this stage, the aim is minimizing the square of error function and obtaining unknown parameters.

F. consider 
$$\sum e^2 = \sum_{k=0}^{N} (g(kT_s) - y(kT_s))^2$$
 as a fitness

function and using Genetic Algorithm Optimization methods for minimizing fitness function.

*Note.* in this method according to transient excitation, assume the settling time is definite and equals to  $t_s$ , then  $t_s = NT_s$ , so N will be obtained if the sampling frequency is definite.(  $f_s = T_s^{-1}$ ) ( $t_s = .05$  in figure 5)

### III. PROBLEM STATEMENT

We want to simulate any DC Motor with estimating the dynamic state of DC Motor. The state equations of the DC Motor system are written as follow [4].

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a}i_a(t) - \frac{K_b}{L_a}\omega_m(t) + \frac{1}{L_a}e_a(t)$$
(7)

$$\frac{d\omega_m(t)}{dt} = \frac{K_T}{J_m} i_a(t) - \frac{B_m}{J_m} \omega_m(t) - \frac{1}{J_m} T_L(t)$$
(8)

Where  $i_a(t)$  is armature current, and  $\omega_m(t)$  is rotor angular velocity  $R_a, L_a, K_b, K_T, B_m, J_m$  are Motors parameters which described in table 1.

According to [4] dynamic treatment of DC Motor can be modeled as follow:

$$\frac{X(s)}{s^2 + 2\eta\omega_n s + \omega_n^2} \qquad Y(s)$$

Fig. 3. Second order dynamic model

where the parameter  $\eta$  is referred to as the damping ratio and the parameter  $\omega_n$  as the un damped natural frequency and **k** is the gain of system. We want to identify these three parameters with minimizing the sum of squared errors in following model:



Fig. 4. Parameter Estimation Block Diagram

Now consider a tested DC Motor: TABLE I TESTED MOTOR PARAMETERS

Resistance	0.8 Ω	Torque	8×10 <sup>-3</sup> V/rpm
R <sub>a</sub>		constant	
<i>na</i>		K <sub>T</sub>	
Inductance	$5 \times 10^{-3} H$	Rotor	1.5×10 <sup>-5</sup> N.M.S
L		interia J	
Back-emf		Friction	2.5×10 <sup>-5</sup> N.M / rpm
constant	$8 \times 10^{-3} V / rpm$	coefficient	
K <sub>b</sub>		f	

According to equations (2),(3),(4) and figure(3) we have only one second order section, so equation (9) is as follow :

$$H(s) = H_{20}(s) = \frac{a_{2k}s + a'_{2k}}{s^2 + b_{2k}s + b'_{2k}} = \frac{k\omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2}$$
(9)

(H(s) is as Estimated Model) then equations (10,11) are as follow :

$$h(t) = \left\{ \frac{k\omega_n^2}{s^2 + 2\eta\omega_n s + \omega_n^2} \right\}^{-1} = \frac{k\omega_n}{\sqrt{1 - \eta^2}} e^{-\eta\omega_n t} \sin(\omega_n \sqrt{1 - \eta^2} t)$$
(10)

$$\hat{y}(t) = \hat{h}(t) * u(t) = k(1 - \frac{1}{\sqrt{1 - \eta^2}} e^{-\eta \omega t} \sin(\omega_n \sqrt{1 - \eta^2} t + \cos^{-1} \eta)$$
(11)

As a result  $\sum e^2 = \sum_{k=0}^{N} (y(kT_s) - \hat{y}(kT_s))^2$  (12)

is considered as fitness function. Consider the response of tested DC Motor to step function:



Now we use Genetic Algorithm methods and obtain unknown parameters. At the end of this paper LSE Estimation is used in comparison with proposed method. (of course gradient base LSE will be used according to transient excitation.) in this section for simplifying the comparison we consider only in  $\eta$  and **k** as unknown parameters and assume  $\omega_n$  is determined.(assume  $\omega_n = 283.5$ )

## IV. PARAMETER ESTIMATION VIA GENETIC ALGORITHM

The genetic algorithm is a stochastic optimization algorithm that was originally motivated by the mechanisms of natural selection and evolution of genetics. In the following, a parameter estimation algorithm is developed based on GA to estimate the unknown parameters, by carrying out minimization of the sum squared errors in (12).

# A. GA Operators

All GAs are effective when used with its best operations and values of parameters. The following operators are modified due to experimental results.

*1)* The fitness function is considered as (12) that must be optimized .for standard optimization algorithm this is known as the objective function.

2) The population size determines the size of the population at each generation. choosing the population size as 50 will be satisfied the results.

this optimization includes two variables so the population can be represented by a 50 by 2 matrix, at each iteration, the genetic algorithm performs a series of computational on the current population to produce a new population.

the algorithm begins by creating a random initial population in interval of [0,2.5] as initial range.

3) At each step , the genetic algorithm uses the current population to create the children that make up the next generation. algorithm usually selects individuals that have better fitness value.

Stochastic uniform used as selection mechanism, it is robust and simple.

4) Elite children are the number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation. elite count considered in this paper is 5 or 10% of population size.

The termination criterion is reaching at 300<sup>th</sup> generation that means algorithm is repeated until the number of generations equal to 300.

5) Gaussian mutation step size and Heuristic crossover used to produce offspring for the next generation. as known a Gaussian mutation operator requires two parameters: the mean, which is often set to zero, and the standard deviation  $\delta$ .  $\delta$  =5.5 in this optimization.

Suppose a child is considered as a value will be produced in the next generation and parents are the values which obtained in previous generation, Heuristic crossover returns a child that lies on the line containing the two parents, a small distance away from the parent with the better fitness value in the direction away from the parent with the worse fitness value.

The parameter Ratio can specify how far the child is from the better parent. The following equation illustrates the relation between parameter Ratio and child (as next generation). Child= parent 2 + R (parent 1- parent 2)

Where parent 1 & parent 2 are the parents, and of course parent 1 has the better value, and R is the parameter Ratio. R=1.2 is considered for second order dynamic model.

6) Using Hybrid function increases the robustness of genetic algorithm. A hybrid function is an optimization function that runs after the genetic algorithm terminates in order to improve the value of fitness function. The Hybrid function uses the final point from the genetic algorithm as its initial point.

We use the function ((fminsearch)) an un constrained minimization function in the optimization,((fminsearch)) uses the simplex search method of [5]. This is a direct search method that does not use numerical or analytic gradients. The results of GA Estimation are collected in table(2)

## B. LSE Estimation

(9) can be written in time domain as follow:

$$\frac{d^{2}y(t)}{dt^{2}} + 2\eta\omega_{n}\frac{dy(t)}{dt} + \omega_{n}^{2}y(t) = k\omega_{n}^{2}x(t)$$
(13)

after using gradient base LSE [1] (consider Appendix)  $\eta$  and **k** will be obtained. The results of LSE Estimation are collected in table(3)

## V. SIMULATION RESULTS

In each estimation three states are considered, and each state includes estimated parameters in corresponded condition and the figures of estimated model according to estimated parameters and the error of this estimated model and tested Motor according to figure(4)

GENETIC ESTIMATION RESULTS					
State	Sampling	Number	Estimated	Estimated	Gain
No.	Time sec	of Data	Parameter	Parameter	error
	Ts	points	k	η	%
		N		-	
1	0.0025	20	12.67373	0.3291	0.0046%
2	0.00125	40	12.66903	0.3142	0.0325%
3	0.0005	100	12.66502	0.3124	0.0642%
			$\begin{array}{c} 0.4 \\ 0.3 \\ - \\ - \\ 0.2 \\ + \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ -$		

Fig. 6. state1: 1) step response of estimated model, 2) error





Fig. 8 . state3 :1) step response of estimated model, 2) error

TABLE III LSE ESTIMATION RESULTS

State	Sampling	Data	Estimated	Estimated	Gain
No.	Time sec	Points	Parameter	Parameter	error
	Ts	Ν	k	$\eta$	%
				•	
1	0.0001	500	12.6730	0.3244	0.0012
2	0.00005	1000	12.6731	0.3174	4.0489e-4
3	0.00001	5000	12.6731	0.3117	4.0489e-4



Fig. 9, state1:1) step response of estimated model, 2) error



Fig. 10 . state2 :1) step response of estimated model, 2) error

TABLE II ENETIC ESTIMATION RESULTS



Fig .11 . state3 :1) step response of estimated model, 2) error

# VI. GA-ESTIMATION IN COMPARISON WITH LSE-ESTIMATION

the difference between GA Estimation and LSE Estimation will be remarkable when the number of data points decrease . the results of simulation illustrate that , high resolution in estimation using LSE method will be obtained since increasing data point numbers and decreasing sampling time or increasing the frequency of sampling. with same data point numbers , the robustness of GA Estimation can be compared with LSE Estimation , figures (12,13,14,15) indicate the resolution of estimation using GA & LSE methods with N=20 , N=40. Tables (4,5) show the resolution of each method.

Remark 1 : Gain error percentage is described as

Gain error %=  $\left| \frac{estimated \ gain - main \ gain}{main \ gain} \right| \%$ 

Remark 2: shaping factor can be obtained as:

Resolution Factor =  $(1 - \frac{\eta_{estimated}}{\eta_{main}}) \times 100\%$ 

Note: of course  $\eta_{main}$  and main Gain are not available but we can assume( only and only for comparison) that DC Motor parameters in table(1) are available

TABLE IV comparison between GA & LSE methods for N=20, 40

Estimation method	Sampling Time sec T <sub>s</sub>	Number of Data points N	Estimated Parameter k
1-GA 2-LSE	0.0025	20	12.67373 12.6835
1-GA 2-LSE	0.00125	40	12.66903 12.6732

TABLE V comparison the resolution of two methods for N=20, 40  $\,$ 

Estimation method	Number of Data points N	Estimated Parameter $\eta$	Resolution Percentage %
1-GA	20	0.3291	93.94 %
2-LSE		0.6540	10.76 %
1-GA	40	0.3142	98.74 %
2-LSE		0.4825	44.50 %









Fig. 14. GA Estimation for N=40(resolution =98.74%)



Fig. 15. LSE Estimation for N=40(resolution =44.50%)

# Conclusion

First, the dynamic state of DC Motor was described with two methods: 1- LSE estimation 2- GA estimation.

Second, according to robustness of GA optimization in finding global optimum and the requirements to high resolution in parameter estimation, this proposed method is applicable in off line parameter estimation.

Third, this method provides accurate estimates of parameters since the number of data points are not as many as the number of data points in LSE estimation. Accurate estimation of the parameters, especially the parameter  $\eta$  which is important in identifying the dynamic state of model, is satisfied with this method.

Fourth, despite of LSE estimation, GA estimation can be used for any LTI systems which are not linear due to parameters.

Appendix (consider [1] pages 60-63) for  $T_s << 1$ 

 $\frac{dy(t)}{dt} = \frac{y(t) - y(t - T_s)}{T_s}$ (for using LSE, (13) must be

discretized, as a result this simple approximation is used, but in general form any other approximations can be used) by substituting in (13):

$$y(t) - 2y(t - T) + y(t - 2T) + \omega_n^2 T^2 y(t - 2T) +$$

$$2\eta \omega_n T [y(t - T) - y(t - 2T)] = k\omega_n^2 T^2 x(t - 2T) \qquad T = T_s$$

$$\omega_n T_s = \alpha \quad so \ the \ model \ structure \ can \ be \ written \ as \ follow:$$

$$\Psi(t) = \varphi^T(t)\theta$$

$$\Psi(t) = y(t) - 2y(t - T) + y(t - 2T) + \alpha^2 y(t - 2T),$$

$$\Psi(t) \ is \ measurable \ quantity.$$

$$\varphi^T(t) = [2\alpha(y(t - 2T) - y(t - T)) \qquad \alpha^2 x(t - 2T)]$$

$$\rho(t) = [\eta] \quad \rho(t) \ is \ a \ two \ vector \ of \ written \ a \ respectvectors.$$

 $\theta(t) = \begin{vmatrix} t \\ k \end{vmatrix}$ ,  $\theta(t)$  is a two vector of unknown parameters.

 $\hat{\theta} = \left[\sum_{t=1}^{N} \varphi(t) \varphi^{T}(t)\right]^{-1} \left[\sum_{t=1}^{N} \varphi(t) \Psi(t)\right] \text{ as a result unknown}$ parameters will be obtained(consider table(3).

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