Deconvolution of Non-Minimum Phase FIR Systems using Adaptive Filtering

M. Lankarany and M.H. Savoji

Abstract— We address, in this paper, the problem of estimating the input sequence of a known, non-minimum phase, FIR system, when a large number of its roots are located near or on the unit circle. This issue cannot be solved by conventional methods known to date. Recently, algorithms based on spectral factorization are considered as possible solutions of inversing non-minimum phase systems but, these techniques cannot prohibit the instability of the systems whose roots are located on the unit circle. We propose an alternative method based on adaptive filtering resulted from a new point of view of the deconvolution problem that avoids inversing the system. The LMS adaptive filter is used to meet our objective while faster implementation than optimization-based techniques, be it gradient based or genetic, is achieved. Moreover, the technique is validated by experimental results, in simulated cases, which are mainly focused on large sequence of signals in noisy conditions.

Index Terms— Deconvolution, Non-Minimum Phase FIR System, LMS Adaptive Filter, Gradient based Adaptive Filter.

I. INTRODUCTION

econvolution is an intricate problem for at least two reasons: the measurements are usually noise corrupted, and the system is frequently non-minimum phase [1]. A simple filtering solution i.e. an inverse system is difficult to find because of these restrictions whose severity depends on the application. The main problem resides in inversing a nonminimum phase system which may cause instability in the deconvolving filter (some poles of the inverse system may be located outside the unit circle in the z-plane). However, deconvolution has a wide range of applications including seismology, channel equalization, numerical differentiation, and speech synthesis [1], [2], [3], [4], [5], [6], [7], and [8]. In [1], the problem of estimating the input to a known linear system is treated in a shift operator polynomial formulation. The mean-square estimation error is minimized. The input and a colored measurement noise are described by independent ARMA (autoregressive moving average) processes. The filter is calculated by performing a spectral factorization and solving a polynomial equation. The approach can be applied to input prediction, filtering, and smoothing problems as well as

to the use of pre-filters in the quadratic criterion.

Applications of deconvolution can be approached with different methods such as Kalman filtering [9], Wiener filtering [11], or Wiener optimization of filters with predetermined structure such as FIR filters [10]. Spectral factorization based algorithms have solved the problem of instability of inverse filtering occurred when the roots are located outside the unit circle but, the problem still remains when these roots are located on or near the unit circle [12].

In this paper, we consider this problem by proposing a new viewpoint of the deconvolution problem which permits avoiding the inverse filtering and instability altogether. The organization of this paper is as follows: in section II a new insight of the deconvolution problem is discussed. In section III and IV an algorithm based on adaptive filtering is presented with two alternative methods for its implementation. Simulation results are reported in section V. Finally, we conclude in section VI by selecting the best method on the basis of speed of computation and accuracy of performance.

II. NEW VIEWPOINT OF DECONVOLUTION PROBLEM

Figure 1.a shows the general illustration for a non-minimum phase LTI systems. Figure 1.b indicates the view point of previous works in the deconvolution problem and figure 1.c shows a new viewpoint which forms the basis of this paper.



Fig. 1. (a). Non-minimum phase system, (b). Viewpoint of previous works used in inverse filter design and, (c). Our proposed viewpoint.

The difference between two points of view resides in the fact that in the previous approaches an inverse filter is sought. This approach runs in difficulties when the roots of the system lie near or on the unit circle in the z-plane. This is while the insight shown in figure 1.c avoids inversing the non-minimum phase system. In other word, the problem is here converted as how to estimate the input signal. This estimation is carried out by modifying the input sequence in order to satisfy a criterion.

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The used criterion is minimizing the sum of squared differences between the original output and the output yielded by the known system whose inverse is sought.

III. PROPOSED ALGORITHM BASED ON ADAPTIVE FILTERING

In [13] a method based on genetic algorithm with a similar viewpoint is used. The speed and accuracy problems encountered in optimization methods, like genetic algorithm, increase rapidly with the number of the unknown parameters. This problem is challenged in [13] by proposing a method which does not need to estimate all parameters jointly. Nevertheless, this method, being time consuming, is not very useful in real time signal processing. In this paper we use adaptive filtering techniques for estimating the sequence of input signal based on our previously mentioned viewpoint of deconvolution. In brief, assuming that the original system is LTI it is allowed replacing the unknown input and the impulse response, in our solution, following the convolution theory. This inverts the role of the input and the impulse response whose estimation is usually aimed at in adaptive filtering. Figure 2 shows how to use an adaptive filter for deconvolution.



Fig. 2. Block diagram of an adaptive filter (up). Setup used for deconvolution by inverting input and impulse response (down).

In the setup used, $\{y(n)\}_{n=1}^{N}$ is the unknown input sequence where N denotes the length of the adaptive filter's input, output and the desired signal. Here, $[w(1), \dots, w(p)]$ is the known finite impulse response of the non-minimum phase system of order p. The desired signal which may be contaminated by additive Gaussian white noise is actually the output of the original system at hand. It is clear that a sequence of zeros should be added to the known impulse response, now being used as input, to make it equal length as the desired signal d(n).

It is noted that in the proposed solution, the impulse response of the adaptive filter is as long as the original system output and the convergence can only be achieved by iterating the algorithm as many times as necessary on the available data. The convergence issue is not dealt with here. Nonetheless, no problem appeared in our simulated test signals of thousand samples long although, as will be mentioned below, the minimum number of iterations used was ten.

In the following two solutions are proposed for estimating the impulse response of the adaptive filter.

A. LMS Adaptive Filtering

The filter coefficients are updated in the LMS adaptive filter [14] as follows.

$$\vec{y}(n) = \vec{y}(n-1) + \mu(n)e(n)\vec{w}(n)$$
(1)

Where

$$\vec{y}(n) = [y_1(n)\cdots y_N(n)]$$

$$e(n) = d(n) - x(n)$$

$$x(n) = \vec{y}(n)\vec{w}(n)^T$$

$$\vec{w}(n) = [w(n)\cdots w(n-p+1)]$$
In which
$$(2)$$

$$\mu(n) = \frac{0 < M < 1}{N \sigma_n^2 / 2}$$

$$\sigma_n^2 = \alpha w^2(n) + (1 - \alpha) \sigma_{n-1}^2$$
(3)

In above, α is a value in the interval between [0, 1].

B. Gradient based method

In the gradient descent algorithm we minimize a fitness function J iteratively starting at some initial point (here an estimate vector of the input signal). The gradient of J is computed at this point, and then moving in the direction of negative gradient or steepest descent by a suitable distance a new point or estimate is arrived at. This procedure is repeated at the new point. According to figure 2, for iterations i=1, 2... the update rule is:

$$\vec{y}(i) = \vec{y}(i-1) - \mu(i)\frac{\partial J}{\partial y}$$
(4)

The fitness function (J) is defined as:

$$J = \sum e^{2}(n) = \sum_{n=1}^{N} [d(n) - x(n)]^{2}$$
(5)

And

$$x(n) = \sum_{n-k=1}^{p} y(k)w(n-k)$$
(6)

$$\frac{\partial J}{\partial y_k} = 2 \sum_{n=1}^{N} [x(n) - \sum_{n-k=1}^{p} y(k)w(n-k)] \frac{\partial x(n)}{\partial y_k}$$
(7)

Where

$$\frac{\partial x(n)}{\partial y_k} = \begin{cases} w(n-k) & \text{if } 1 \le n-k \le p \\ 0 & \text{if others} \end{cases}$$
(8)

And compute $\frac{\partial \vec{J}}{\partial v}$ is formed as:

$$\frac{\partial \vec{J}}{\partial y} = \left[\frac{\partial J}{\partial y_1} \cdots \frac{\partial J}{\partial y_N}\right] \tag{9}$$

By replacing (9) in (4), the updated value of $\vec{y}(i)$ or the new input estimate is calculated. In the above formulation $\mu(i)$

is the step size at iteration i and is a constant equal to 0.001 in our case.

It is noted that in the gradient descent algorithm, the exact gradient of the mean square error is used whilst in the LMS algorithm a noisy estimate is employed. As the whole available data is used every time the exact gradient is calculated, the gradient descent algorithm is blocked based and consumes a much higher computation time than the LMS algorithm which is sample based and sequential.

IV. STEP BY STEP DESCRIPTION OF PROPOSED ALGORITHM

The description of our algorithm (using both methods) is shown in the following block diagrams. Figure 3 depicts the deconvolution problem for a non-minimum phase system with the shown impulse response where the unknown input must be estimated.



Fig. 3. Deconvolution problem where input is to be estimated

Note that we assume that the output and unknown input signal have the same length. Command "filter" in MATLAB is used to generate the output.

Figure 4 shows how the deconvolution problem is converted into an adaptive filter impulse response calculation.



Fig. 4. Using adaptive filtering for estimating the unknown input

Now, the desired and the input signals of the above adaptive

filter are repeated at least 10 times to provide reliable estimation of the filter coefficients. Figure 5 is a graphic description of our algorithm.



Fig. 5. Estimation of the unknown input using adaptive filter and repeated signals.

V. SIMULATION RESULTS

A large number of simulations for different types of nonminimum phase FIR systems are tested with different signal lengths using our proposed algorithm. Convergence was achieved in all cases. In the following simulation results of only two cases are discussed as examples. Both mentioned methods of LMS and Gradient based Adaptive Filter are evaluated in each example. The original input is checked for accuracy with its estimate. The output is calculated by convolving the true input signal with the impulse response of the non-minimum phase FIR system. Since the simulation involves a non-minimum phase system whose roots (zeroes) lie mostly near or on the unit circle for which other solutions fail, no comparison is made with other alternatives.

Experiment 1: In this example we use a long sequence of a speech signal corresponding to vowel 'a' as the original input assumed unknown in our deconvolution problem. Figure 6 shows this input, the impulse response and the noise corrupted output signal. A Gaussian noise is added to the original output at 10dB Signal to Noise Ratio (SNR). Figure 7 is the plots of estimated versus original input as calculated using our proposed methods. The non-minimum phase system impulse response is calculated using "firgr" command of MATLAB filter design toolbox. This routine uses the Parks-McClellan technique to design digital FIR filters. The input parameters of the routine are chosen by trial and error to make sure that a desired system with enough number of zeros near and on the unit circle is designed.



Fig. 6. (A). Original input, (B). Zero plot of system, (C). Impulse response and (D). Noise corrupted output



Fig. 7. (A). Estimated input using LMS adaptive filtering (B). LMS estimated versus original input, (c). Estimated input using Gradient based optimization and (D). Gradient based estimated versus original

Experiment 2: The original input sequence in this example is chosen to be similar to a seismic reflection signal. Figure 8 shows the original input, the impulse response and the noise corrupted output signal. A Gaussian noise (SNR 20dB) is added to the original output. Figure 9 shows the comparative results for this case.



Fig. 8. (A). Original input, (B). Zero plot of system, (C). Impulse response and (D). Noise corrupted output



Fig. 9. (A). Estimated input using LMS adaptive filtering (B). LMS estimated versus original input, (c). Estimated input using Gradient based optimization and (D). Gradient based estimated versus original

The obtained results can be compared more objectively

using the error percentage defined as follows:

$$Error = \sqrt{\frac{\sum_{n} (original input - estimated input)^{2}}{\sum_{n} (original input)^{2}} \times 100\%}$$

Table 1 and 2 shows the comparison between the proposed methods, in terms of error percentage and computation time, for experiment 1 and 2 respectively.

Table 1: Comparison between two proposed methods - experiment

1			
Method	Error Percentage	Computation Time	
LMS Adaptive Filter	2.03%	4.00 seconds	
Gradient Based Adaptive Filter	3.46%	123 seconds	

Table 2: Comparison between two proposed methods - experiment

<u>2</u>			
Method	Error	Computation Time	
	Percentage		
LMS Adaptive Filter	4.18%	1.05 seconds	
Gradient Based Adaptive Filter	7.00%	43 seconds	

VI. CONCLUSION

Two novel methods based on adaptive filtering have been proposed, in this work, for deconvolution of non-minimum phase FIR systems. These methods permit estimating the unknown input sequence while avoiding inversion of nonminimum phase systems. Based on our proposed algorithm the unknown input sequence is identified as the coefficients of an adaptive filter while the original impulse response, after zero padding, is used as the input. Also, unlike other methods; ours is not dependent on the system type (non/minimum phase), output length and noisy conditions. Experimental results have shown that the LMS method performs better, in terms of speed and accuracy, than the Gradient based adaptive filtering.

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